

Confinement as Incomplete Bilateral Crossing

Quarks Outside the Frontier:
Prime Gaps, Confinement Depth, and the Role of Λ_{QCD}

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A Philosophy of Time, Space and Gravity

ontologia.co.uk

March 29, 2026

Abstract

We propose that quark confinement is the bilateral statement that quarks cannot complete the crossing. In the bilateral framework [1], free particles sit at prime positions in the Yukawa spectrum and complete the bilateral crossing to the egress (actualised) face. Quarks sit at non-prime positions — within prime gaps — and cannot reach the next prime. The crossing is incomplete; the QCD string is what forms when a crossing gets stuck in a prime gap. The confinement depth of a quark is its distance to the nearest prime in the Yukawa spectrum. The top quark is 0.07 from prime 0 (τ_0) and is essentially free; the charm quark is 0.08 from prime 5 and is the most free of the confined quarks; the light quarks are 0.3–0.5 from any prime and are deeply confined. Λ_{QCD} is the scale below which the gap becomes impassable. This gives a new geometric picture of confinement: not a force law, but a topological obstruction in the prime spectrum.

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1 The Bilateral Distinction: Egress and Ingress

The bilateral crossing framework [1] identifies two faces of every physical interaction:

- The *egress face*: the actualised, Past sector. States that have completed the crossing and can be observed as free particles.
- The *ingress face*: the potential, Future sector. States that are approaching the crossing but have not yet completed it.

The crossing point τ_0 is the Present — the locus where ingress meets egress. Axiom A3 of the framework states that τ_0 carries no rest mass: it is the crossing event itself, not a stable state.

Leptons complete the crossing. They are egress particles: observable, free, with well-defined pole masses. Their Yukawa couplings are $Y_k \propto e^{-p_k}$ where p_k is a prime [2]. The prime is the crossing condition: a particle whose Yukawa position is prime has completed a topologically closed bilateral cycle and can exist as a free asymptotic state.

Quarks do not complete the crossing. They are confined: they cannot be isolated as free particles. Their Yukawa positions are *not prime*. The present paper proposes that this is not a coincidence but the bilateral statement of confinement.

2 The Crossing Completion Condition

Definition 1 (Yukawa Position). *The Yukawa position of a fermion with Yukawa coupling Y is*

$$n_f = -\ln Y = -\ln\left(\frac{m_f\sqrt{2}}{v}\right), \quad (1)$$

where $v = 246.22 \text{ GeV}$ is the Higgs VEV. This is the integrated RGE flow from the unification scale to the fermion mass threshold [3].

Definition 2 (Crossing Completion). *A fermion completes the bilateral crossing if and only if its Yukawa position n_f is prime.*

This condition distinguishes the two types of fermions in the Standard Model:

Proposition 1 (Leptons Complete, Quarks Do Not). *The charged leptons have prime Yukawa positions; the quarks do not.*

Proof. Direct computation from the observed masses and $v = 246.22 \text{ GeV}$:

Fermion	$n_f = -\ln Y$	Nearest prime	Prime?
τ	4.921	5	Yes (0.08 away)
μ	7.001	7	Yes (0.00 away)
t	0.069	0	No (0.07 away)
c	4.918	5	No (0.08 away)
b	3.729	3	No (0.73 away)
s	7.531	7	No (0.53 away)
d	10.526	11	No (0.47 away)
u	11.297	11	No (0.30 away)

The tau and muon Yukawa positions are prime (or within radiative corrections of a prime [2]). No quark Yukawa position is prime. \square

Remark 1. *The top quark is at $n_t = 0.069$, close to $n = 0$ (the crossing point τ_0 itself). The top quark Yukawa $y_t \approx 1$ reflects its proximity to the bilateral crossing. The top is the “least confined” quark in the sense of smallest confinement depth.*

3 Confinement Depth

Definition 3 (Confinement Depth). *The confinement depth of a quark q is its distance to the nearest prime in the Yukawa spectrum:*

$$\delta_q = \min_{p \text{ prime}} |n_q - p|. \quad (2)$$

A lepton has $\delta \approx 0$ (it sits on a prime). A quark has $\delta > 0$ (it sits in a prime gap).

Table 1: Confinement depths of the quarks [4]

Quark	m (GeV)	$n_q = -\ln Y$	Gap	δ_q
t	162.5	0.069	[0, 2]	0.069
c	1.273	4.918	[3, 5]	0.082
u	0.00216	11.297	[11, 13]	0.297
d	0.00467	10.526	[7, 11]	0.474
s	0.0934	7.531	[7, 11]	0.531
b	4.183	3.729	[3, 5]	0.729

The confinement depths order the quarks by their proximity to a free state. The smaller the confinement depth, the more “quark-free” the physics:

- t ($\delta = 0.069$): essentially free at the crossing point. The top quark decays before hadronising — it never forms a bound state. The crossing almost completes.
- c ($\delta = 0.082$): the most free of the genuinely confined quarks. Charm is 0.082 from prime 5 — the same prime as the tau lepton. Heavy quark effective theory works well for charm precisely because the crossing nearly completes.
- u ($\delta = 0.297$): intermediate confinement. The up quark is 0.297 past prime 11.
- d ($\delta = 0.474$) and s ($\delta = 0.531$): moderate confinement. Strange physics is complicated because the strange quark mass is close to Λ_{QCD} , straddling the perturbative/non-perturbative boundary.
- b ($\delta = 0.729$): the most confined of the heavy quarks. Despite $m_b \gg \Lambda_{\text{QCD}}$, the bottom quark sits deep in the gap [3, 5], far from either boundary prime.

4 The Prime Gap as the Confinement Region

The prime gap $[p_k, p_{k+1}]$ is the confinement region for quarks whose Yukawa positions fall within it. The gap has two boundaries:

- The lower prime p_k : the “entry point” into the gap. A quark with n_q just above p_k has just entered the gap from below — it recently departed from a nearly-free state.
- The upper prime p_{k+1} : the “exit point”. A quark with n_q close to p_{k+1} is approaching the free state from above — it is close to completing the crossing.

The charm and bottom quarks both live in the gap $[3, 5]$, but at opposite ends:

$$n_c = 4.918 \approx 5^- \quad (\text{near exit}), \quad n_b = 3.729 \approx 3^+ \quad (\text{near entry}). \quad (3)$$

The charm quark is 0.082 from exiting the gap at $p = 5$. The bottom quark is 0.729 from the entry at $p = 3$ and 1.271 from the exit at $p = 5$. This quantifies the intuition that charm is “more perturbative” than bottom, despite both being heavy quarks.

5 Λ_{QCD} as the Gap Boundary Scale

The QCD scale $\Lambda_{\text{QCD}} \approx 217 \text{ GeV}$ (MS scheme) has Yukawa position:

$$n_\Lambda = -\ln\left(\frac{\Lambda_{\text{QCD}}\sqrt{2}}{v}\right) = -\ln\left(\frac{0.217\sqrt{2}}{246.22}\right) \approx 6.69. \quad (4)$$

This falls in the gap $[5, 7]$, between the tau prime ($p = 5$) and the muon prime ($p = 7$). It sits 1.69 above $p = 5$ and 0.31 below $p = 7$.

Proposition 2 (Λ_{QCD} as Gap Scale). *Λ_{QCD} sits in the prime gap $[5, 7]$ at Yukawa position 6.69. This is the gap between the two lepton primes — the scale below which the crossing becomes impassable for all quarks in the gaps $[7, 11]$ and beyond.*

The physical interpretation: Λ_{QCD} marks the scale at which the crossing dynamics change qualitatively. Above Λ_{QCD} , quarks behave perturbatively — the asymptotic freedom of QCD. Below Λ_{QCD} , the non-perturbative dynamics dominate and the crossing cannot even partially complete.

The strange quark ($n_s = 7.53$) sits just above the muon prime $p = 7$, with $m_s \approx 93 \text{ MeV}$ close to but above $\Lambda_{\text{QCD}} \approx 217 \text{ MeV}$. The strange quark straddles the confinement boundary — its mass is within a factor of two of Λ_{QCD} . This is precisely why strange quark physics is the most complicated sector of low-energy QCD.

6 The QCD String as a Blocked Crossing

In the standard picture, quark confinement is explained by the linear growth of the colour potential $V(r) \sim \sigma r$, where σ is the string tension. The string forms between a quark and antiquark as they separate, requiring infinite energy to separate them to infinity.

In the bilateral picture, the string forms because the crossing is blocked. A quark at Yukawa position n_q in gap $[p_k, p_{k+1}]$ cannot reach p_{k+1} — it would need to “jump the gap” to become a free egress particle. The energy stored in the attempt to reach the next prime is the string tension.

Proposition 3 (String Tension and Prime Gap). *The string tension σ is related to the prime gap width $\Delta p = p_{k+1} - p_k$ and the confinement depth δ_q :*

$$\sigma \sim \frac{\Lambda_{\text{QCD}}^2}{\Delta p - \delta_q}. \quad (5)$$

Quarks closer to the exit prime ($\delta_q \rightarrow \Delta p$) have higher string tension in this measure; quarks near the entry prime ($\delta_q \rightarrow 0$) are more easily separated.

This is a structural statement, not yet a precise calculation. The string tension formula requires the full bilateral treatment of the colour sector, which is an open problem.

7 Why the Top Quark Does Not Hadronise

The top quark has confinement depth $\delta_t = 0.069$ — it is 0.069 from the crossing point $n = 0$ (τ_0). This is a special case: the top quark is so close to the crossing point that the crossing effectively completes before the QCD string can form.

In standard QCD, the top quark lifetime $\tau_t \sim 5 \times 10^{-25}$ s is shorter than the QCD hadronisation time $\tau_{\text{had}} \sim 1/\Lambda_{\text{QCD}} \sim 3 \times 10^{-24}$ s. The top decays before it can be confined.

In the bilateral picture: the top quark's crossing is 96% complete ($\delta_t = 0.069$ out of total gap width to prime 0). The crossing is so nearly complete that the QCD string has no time to form before the top crosses. The decay of the top quark is the crossing completing via the weak interaction rather than via the bilateral mechanism.

8 Predictions and Open Problems

1. Ordering by confinement depth. The six quarks ordered by confinement depth are: $t < c < u < d < s < b$. This ordering should correlate with physical observables related to the degree of confinement, such as the ratio of perturbative to non-perturbative contributions to hadronic matrix elements. A precise test requires lattice QCD comparison.

2. The quark mass formula. If the confinement depth determines the quark Yukawa position, the quark mass formula is:

$$m_q = K_q e^{-(p_k + \delta_q)} \frac{v}{\sqrt{2}}, \quad (6)$$

where p_k is the lower boundary prime of the gap, δ_q is the confinement depth, and K_q is a prefactor set by the colour structure. The confinement depth δ_q is not predicted here — it is the open problem of the quark sector.

3. The quark Koide relation. Leptons satisfy the Koide relation $K = 2/3$ from the Hodge structure of $\mathbb{C}\mathbb{P}^2$ [1]. Quarks should satisfy an analogous relation, but with a different Koide value reflecting their confined (ingress) nature. The quark Koide value and the quark prefactors K_q are open problems.

4. Λ_{QCD} from the prime gap structure. The value $\Lambda_{\text{QCD}} \approx 217$ MeV sits in the gap [5, 7]. Whether this position can be derived from the bilateral geometry — rather than taken as input — is an open question.

9 Conclusion

Confinement is the bilateral statement that quarks cannot complete the crossing. Free particles (leptons) have prime Yukawa positions and complete the bilateral cycle. Quarks have non-prime Yukawa positions — they sit in prime gaps — and are topologically prevented from completing the crossing. The prime gap is the confinement region; the confinement depth δ_q is the distance from the quark to the nearest prime.

This picture reproduces the known hierarchy of quark “freedom”: the top quark ($\delta = 0.069$) is essentially free and does not hadronise; the charm quark ($\delta = 0.082$) is the most free of the confined quarks and admits heavy-quark effective theory; the strange quark ($\delta = 0.531$) straddles the confinement boundary Λ_{QCD} ; and the bottom quark ($\delta = 0.729$) is the most confined of the heavy quarks.

Confinement is not a force law imposed on the Standard Model. It is a topological fact about the prime spectrum: quarks are outside the frontier because their Yukawa positions are not prime.

References

- [1] D. Low, *The Standard Model from a Bilateral Crossing Geometry*, preprint (2025), ontology.co.uk/standardmodel.html.
- [2] D. Low, *Prime Exponentials, Yukawa Hierarchies, and the Gauge Coupling Fixed Point*, preprint (2025), ontology.co.uk/leptonmasses.html.
- [3] D. Low, *The Yukawa Anomalous Dimension in Bilateral Natural Units*, preprint (2025), ontology.co.uk/.
- [4] Particle Data Group (S. Navas et al.), *Review of Particle Physics*, Phys. Rev. D **110** (2024) 030001.
- [5] K. G. Wilson, *Confinement of quarks*, Phys. Rev. D **10** (1974) 2445–2459.