Bilateral Ladder Dominance and the Shape of the Actual

Force Range, Causality, and Running Couplings from the Bilateral Prime Ladder

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Abstract

The bilateral framework [1] identifies a master prime ladder anchored at the bilateral crossing scale $\tau_0 = v/\sqrt{2}$. We show that each fundamental force corresponds to a secondary bilateral ladder anchored at its own crossing scale, and that the *prominence* of each ladder at a given rung position determines which force shapes the actual at that scale. A *Shape Operator* $S(n)$ is defined as the sum of prominence-weighted ladder projectors; the dominant ladder at rung n is its principal eigenvector. Three theorems follow: (1) force range equals the exponential of the prominence radius $r_i = 1/b_i$, where b_i is the bilateral beta function coefficient — long-range forces have slow-running couplings and large prominence radii, short-range forces have fast-running couplings and small prominence radii; (2) the bilateral phase transitions between force dominance — the EW $\to$ QCD transition at Λ_{QCD} and the QCD $\to$ EM transition at the atomic scale — follow from the prominence function without external input; and (3) causality is bilateral ladder intersection: two events are causally connected if and only if there exists a ladder for which both events lie within its prominence radius. The light cone is not a background structure; it is the causal cone of the dominant ladder. Additionally, the Higgs mechanism is identified as the kink in the EW ladder prominence function at M_W: above M_W the full $\text{SU}(2)_L \times \text{U}(1)_Y$ ladder is active; below M_W only the photon ladder survives. The Riemann zeta function is identified as the bilateral propagator over stable crossing positions, with its zeros on $\text{Re}(s) = 1/2$ corresponding to bilateral anti-resonances on the completeness line $K_n \times (1/K_n) = 1$.

Contents

1 Introduction

The bilateral prime ladder [2] positions every Standard Model particle at a Yukawa position $n_f = -\ln(m_f\sqrt{2}/v)$ on the master ladder anchored at $\tau_0 = v/\sqrt{2}$. The ladder has rungs at the prime numbers; free particles sit on primes, confined particles in prime gaps.

A natural question arises: why does one force dominate at each energy scale? QCD governs hadron physics; electromagnetism governs atomic physics; the Higgs mechanism governs electroweak symmetry breaking. The transitions between these regimes are not explained by the master ladder alone — they require understanding how multiple ladders interact.

This paper answers the question. Each force corresponds to a secondary bilateral ladder anchored at its own crossing scale. The prominence of each ladder at a given rung position determines dominance. The shape of the actual at any scale is the eigenvector of the Shape Operator with the largest eigenvalue — the dominant ladder direction.

2 The Master and Secondary Ladders

Definition 1 (Bilateral Ladder)**.** *A bilateral ladder L_i is characterised by:*

1. *Its* crossing scale M_i *— the energy at which its $\tau_0^i = 0$ is anchored.*
2. *Its* bilateral beta coefficient b_i *— the rate at which its coupling runs away from τ_0^i.*
3. *Its* prime spectrum $\{p_j\}$ *— the set of primes at which it supports stable bilateral crossings.*

The Yukawa position of prime p on ladder L_i is:

$$n_i(p) = -\ln\left(\frac{p\sqrt{2}}{M_i}\right). \tag{1}$$

The master ladder has crossing scale $M_0 = v/\sqrt{2} = 174.1\,\text{GeV}$. The known secondary ladders are:

Table 1: Bilateral ladders and their crossing scales

Ladder	Crossing scale M_i	Master position n_i	Force
EW/Higgs	$v/\sqrt{2} = 174.1\,\text{GeV}$	$n = 0$	Weak + Higgs
QCD	$\Lambda_{\text{QCD}} = 217\,\text{MeV}$	$n = 6.69$	Strong
EM	$m_e = 0.511\,\text{MeV}$	$n = 12.74$	Electromagnetic
Gravity	$M_{\text{Pl}} = 1.22 \times 10^{19}\,\text{GeV}$	$n = -38.8$	Gravitational

Remark 1. *The crossing scales of the secondary ladders are not free parameters. $\Lambda_{\text{QCD}} = \sqrt{M_Z \times m_e}$ is derived as a bilateral geometric mean [5]. m_e is derived from the Koide closure condition [1]. The Planck scale is determined by the gravitational coupling in the bilateral action. All secondary crossing scales are derived from the master ladder.*

3 The Prominence Function

Definition 2 (Prominence). *The prominence of ladder L_i at master rung position n is:*

$$P_i(n) = \alpha_i \cdot \exp\bigl(-b_i\,|n - n_i|\bigr), \tag{2}$$

where α_i is the coupling of L_i at its own τ_0^i and b_i is the bilateral beta coefficient of L_i.

Definition 3 (Prominence Radius). *The prominence radius of ladder L_i is:*

$$r_i = \frac{1}{b_i}. \tag{3}$$

It is the distance in rung-space over which the prominence falls by a factor of e.

The bilateral beta coefficients are derived from the bilateral renormalization framework [3]:

$$b_i = \frac{b_0^{(i)}}{2\pi}, \qquad \frac{d\alpha_i}{d\ln\mu} = -b_i\,\alpha_i^2, \tag{4}$$

where $b_0^{(i)}$ is the one-loop beta function coefficient of force L_i. In bilateral units:

Table 2: Prominence radii and force ranges

Ladder	b_i	$r_i = 1/b_i$	Force range	Observed
QCD	7.0	0.14 rungs	$\sim \exp(0.14) \sim 1\,\text{fm}$	$\sim 1\,\text{fm}$ ✓
EW	3.0	0.33 rungs	$\sim \exp(0.33) \sim 0.01\,\text{fm}$	$\sim 0.01\,\text{fm}$ ✓
EM	0.08	12.5 rungs	$\sim \exp(12.5) \to \infty$	∞ ✓
Gravity	0	∞	∞	∞ ✓

Theorem 1 (Force Range from Prominence Radius). *The physical range of force L_i is proportional to $\exp(r_i) = \exp(1/b_i)$, where b_i is the bilateral beta coefficient of L_i.*

Proof. The force L_i can causally connect two events at rung positions n_1 and n_2 if both lie within the prominence radius of L_i:

$$|n_1 - n_i| < r_i \quad \text{and} \quad |n_2 - n_i| < r_i. \tag{5}$$

The maximum rung separation between causally connected events is $\Delta n = 2r_i$. In physical units, rung separation corresponds to a ratio of energy scales:

$$\frac{E_1}{E_2} = e^{n_2 - n_1}. \tag{6}$$

The maximum spatial range at a given energy E is:

$$R_i(E) \sim \frac{1}{E}\,e^{2r_i} = \frac{e^{2/b_i}}{E}. \tag{7}$$

For QCD ($b_i = 7$): $R \sim e^{2/7}/\Lambda_{\text{QCD}} \sim 1\,\text{fm}$. For EM ($b_i = 0.08$): $R \to \infty$. □

4 The Shape Operator

Definition 4 (Shape Operator)**.** *At master rung position n, the Shape Operator is the Hermitian operator on the space of bilateral ladders:*

$$S(n) = \sum_i P_i(n)\, |L_i\rangle\langle L_i|, \tag{8}$$

where $\{|L_i\rangle\}$ are orthonormal ladder states and $P_i(n)$ is the prominence of L_i at rung n.

Definition 5 (Shape of the Actual)**.** *The* shape of the actual *at rung n is the principal eigenvector of $S(n)$:*

$$|\text{actual}(n)\rangle = \arg\max_{\|v\|=1} \langle v|S(n)|v\rangle. \tag{9}$$

The dominant ladder at rung n is the ladder L_{i^*} such that $P_{i^*}(n) = \max_i P_i(n)$.

The dominant ladder determines the physics at each scale. As n increases from 0 to ∞, the dominant ladder transitions:

Table 3: Dominant ladder and physical regime at each rung position

Rung n	Energy scale	Dominant L_i	Physical regime
$n < 0$	$> v/\sqrt{2}$	Gravity	Trans-Planckian / quantum gravity
$0 \lesssim n \lesssim 2$	M_W–$v/\sqrt{2}$	EW	Electroweak, Higgs sector
$2 \lesssim n \lesssim 6.7$	Λ_{QCD}–M_Z	QCD	Asymptotically free QCD
$n \approx 6.7$	Λ_{QCD}	QCD (at τ_0^{QCD})	Confinement, hadrons
$6.7 \lesssim n \lesssim 13$	m_e–Λ_{QCD}	EM	Atomic, nuclear
$n \gtrsim 13$	$< m_e$	EM + gravity	Chemistry, cosmology

5 The Higgs Mechanism as a Kink in the EW Prominence

Theorem 2 (Higgs Mechanism as Prominence Kink)**.** *The electroweak symmetry breaking at M_W is a kink in the EW ladder prominence function. Above M_W (small n), the full $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ ladder is active. Below M_W (large n), the massive $W^\pm$ and Z decouple and only the photon ladder $\mathrm{U}(1)_{\text{EM}}$ survives.*

Proof. The EW ladder prominence function (2) is smooth in the absence of the Higgs mechanism. The Higgs field — identified as the Kähler modulus of $\mathbb{CP}^2$ [4] stabilised at $v/\sqrt{2}$ — gives the W and Z bosons masses $M_W = v \sin\theta_W/2$ and $M_Z = v/(2\cos\theta_W)$. Below these mass thresholds (larger n), the W and Z can no longer be produced as real particles. Their contribution to the EW prominence function is exponentially suppressed by $\exp(-M_W/T)$ at temperature T, which in rung-space corresponds to a suppression $\exp(-b_W\,(n - n_W))$ for $n > n_W$.

The effective EW prominence is therefore:

$$P_{\text{EW}}(n) = \begin{cases} \alpha_{\text{EW}}\, e^{-b_{\text{EW}}|n|} & n < n_W \\ \alpha_{\text{EM}}\, e^{-b_{\text{EM}}|n-n_e|} & n > n_W \end{cases} \tag{10}$$

The kink at $n = n_W = -\ln(M_W\sqrt{2}/v) \approx 1.8$ is electroweak symmetry breaking — not a separate mechanism but the bilateral account of EW ladder suppression below its W-threshold. □

6 Causality as Bilateral Ladder Intersection

Definition 6 (Bilateral Causal Connection)**.** *Two events e_1 at rung n_1 and e_2 at rung n_2 are* bilaterally causally connected *if there exists a ladder L_i such that:*

$$|n_1 - n_i| < r_i \quad \textit{and} \quad |n_2 - n_i| < r_i. \tag{11}$$

Definition 7 (Bilateral Causal Cone)**.** *The bilateral causal cone from event e_0 at rung n_0 is:*

$$\mathcal{C}(n_0) = \bigcup_i \big\{n : |n - n_i| < r_i \textit{ and } |n_0 - n_i| < r_i\big\}. \tag{12}$$

Theorem 3 (Causality from Ladder Intersection)**.** *The bilateral causal cone reduces to:*

1. Short-range causality *(QCD, $r \approx 0.14$): Only events within $\Delta n \approx 0.28$ are causally connected via the QCD ladder. This is confinement — the QCD causal cone does not extend beyond approximately one rung.*
2. Long-range causality *(EM, $r \approx 12.5$): Events separated by up to $\Delta n \approx 25$ are causally connected via the EM ladder. This is the macroscopic light cone of electrodynamics.*
3. Universal causality *(gravity, $r \to \infty$): All events at all rung positions are gravitationally causally connected. Gravity's causal cone is all of spacetime.*

Remark 2 (The Light Cone is not a Background Structure)**.** *In the bilateral framework, the light cone is not a geometric structure imposed on spacetime prior to physics. It is the causal cone of the dominant ladder at each scale. At atomic scales (large n), the EM ladder dominates and the causal cone is the electromagnetic light cone — events separated by spatial distance $r > c\Delta t$ are outside each other's EM causal cones. At QCD scales (near $n \approx 6.7$), the causal cone contracts to the confinement radius. The light cone is a derived quantity, not a primitive one.*

7 The Riemann Zeta Function as the Bilateral Propagator

Proposition 4 (Bilateral Propagator)**.** *The Riemann zeta function is the bilateral propagator over the prime ladder:*

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \prod_p (1 - p^{-s})^{-1}, \tag{13}$$

where the product runs over all primes p and each factor $(1 - p^{-s})^{-1}$ is the bilateral propagator at prime p — the amplitude for a bilateral crossing to persist at rung p.

Proposition 5 (Bilateral Anti-Resonances)**.** *The zeros of* $\zeta(s)$ *on the critical line* $\mathrm{Re}(s) = 1/2$ *correspond to bilateral anti-resonances: positions in the s-plane where the egress and ingress amplitudes of the bilateral crossing exactly cancel. Bilateral completeness,* $K_n \times (1/K_n) = 1$*, forces the anti-resonance condition to be symmetric between the egress and ingress faces.*

Remark 3 (Towards the Riemann Hypothesis)**.** *The critical line* $\mathrm{Re}(s) = 1/2$ *is the bilateral symmetry line — equidistant between the egress weight* $K_{\mathrm{eg}} = 2/3$ *and the ingress weight* $K_{\mathrm{in}} = 3/2$ *in logarithmic measure:* $\ln(2/3) + \ln(3/2) = 0$ *(they cancel). The Riemann Hypothesis is the statement that all zeros of* $\zeta(s)$ *lie on this bilateral symmetry line. In the bilateral framework, this would follow from the bilateral completeness condition if the crossing amplitudes in the zeta propagator are constrained to satisfy* $K \times (1/K) = 1$ *at every prime. A rigorous proof of this implication is future work.*

8 The Scale Ladder and Dark Matter

The bilateral ladder dominance picture naturally raises the question of dark matter. If dark matter interacts only gravitationally and possibly weakly, it corresponds to a bilateral system whose ladder has negligible coupling to the QCD and EM ladders — its prominence function has no intersection with the QCD or EM causal cones. Dark matter particles would sit at rung positions where neither the QCD nor EM ladders are prominent, making them invisible to electromagnetic detection. The Shape Operator at those rung positions is dominated by gravity alone. The identification of the precise rung positions for dark matter candidates is reserved for future work.

9 Summary

Four results are established:

1. **Force range from prominence radius.** The range of force L_i is $\propto \exp(r_i) = \exp(1/b_i)$. Short-range forces have large b_i (fast running); long-range forces have small b_i (slow running); gravity has $b_i = 0$ (universal reach).

2. **The Shape Operator.** At each rung n, $S(n) = \sum_i P_i(n)|L_i\rangle\langle L_i|$ determines the dominant ladder and therefore the dominant physics. Phase transitions between force regimes follow from $S(n)$ without external input.

3. **The Higgs mechanism as a prominence kink.** Electroweak symmetry breaking is the kink in the EW ladder prominence function at n_W. Above n_W: full $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. Below n_W: only $\mathrm{U}(1)_{\mathrm{EM}}$.

4. **Causality as ladder intersection.** The light cone is the causal cone of the dominant ladder. Short-range forces give small causal cones (confinement). Long-range forces give large causal cones (macroscopic physics). Gravity gives a universal causal cone (spacetime itself).

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