Möbius Cascade

Topologically Sustained Weak-Collapse Computation via Half-Möbius Quantum Gate Arrays

Dunstan Low

A Philosophy of Time, Space and Gravity

ontologia.co.uk

May 2026

Abstract

We propose a quantum computational architecture — the Möbius cascade — in which weak measurements are chained around a ring of qubits coupled with a half-Möbius topology (90° phase twist per gate step). In the bilateral mesh framework, a weak measurement is a partial bilateral crossing: it writes a physical egress record — a microwave photon — without fully consuming the ingress superposition. That record, routed to the next qubit, updates its ingress potential and triggers the next weak crossing. The half-Möbius coupling enforces a 720° spinor return condition: the cascade signal must traverse the ring twice before returning to its starting phase, preventing the system from settling into a steady state and sustaining oscillation over timescales significantly longer than individual qubit coherence times. We derive four testable predictions: (1) the output sequence has period $2N$ events; (2) outputs separated by N steps are anti-correlated (the 720° sign flip); (3) the cascade sustains itself above a sharp collapse threshold at crossing angle $\theta_c = \pi/4$; and (4) the cascade terminates abruptly when measurement strength exceeds θ_c, exhibiting a sharp phase transition. These predictions are testable on existing superconducting quantum processors. The architecture is grounded in the bilateral crossing geometry of $S^3 \times \mathbb{CP}^2$ [1] and the experimental realisation of half-Möbius electronic topology in $C_{13}Cl_2$ [4], but the computational proposal stands independently of those foundations.

Contents

1 Introduction

Standard quantum computation proceeds by unitary evolution followed by projective measurement. Measurement is treated as an endpoint: the superposition collapses, the result is read out, and the computational resource is consumed. Correlated decoherence across multiple qubits is treated as a noise source to be suppressed [5].

Two recent experimental results suggest a different perspective. In March 2026, IBM Research and collaborators synthesised $C_{13}Cl_2$, the first molecule exhibiting half-Möbius electronic topology [4]: a 13-membered carbon ring in which the electronic phase twists by exactly 90° per revolution, requiring four complete circuits for phase return. The molecule can be reversibly switched between three topological states — a direct physical realisation of the three bilateral crossing positions (egress, crossing point, ingress).

This experimental result motivates a computational architecture in which topology — rather than coherence — is the primary resource. We propose the *Möbius cascade*: a ring of N qubits coupled with a half-Möbius phase twist, in which weak measurements propagate around the ring and sustain themselves through the 720° spinor return condition. The cascade does not require the superposition to be preserved across all qubits simultaneously; it requires only that each weak measurement write a sufficiently clean egress record to trigger the next. The topological structure of the coupling does the rest.

2 The Bilateral Crossing and Weak Measurement

We briefly recall the relevant structure of the bilateral mesh framework [1, 2].

Definition 2.1 (Bilateral crossing)**.** *A bilateral crossing is an event at the crossing point τ_0 (the present) at which the ingress face (potential, superposition) is partially or fully actualised, writing a record on the egress face (past, actualised). The crossing angle $\theta \in [0, \pi/2]$ parametrises the strength of the crossing:*

- $\theta = \pi/2$*: projective (full) measurement. The ingress potential is entirely consumed. The egress record is maximal. The superposition is destroyed.*
- $\theta \ll \pi/2$*: weak measurement. A fraction $\sin^2\theta$ of the potential is actualised; a fraction $\cos^2\theta$ survives as residual superposition.*
- $\theta = \pi/4$*: the bilateral mid-crossing. Equal actualisation and residual; the facing direction is $e^{i\pi/4}$. This is the threshold crossing angle.*

In physical terms, a weak measurement on a superconducting qubit is implemented by a dispersive readout with a short, low-power coherent tone: the qubit state is partially resolved, a microwave photon is emitted into the readout waveguide (the egress record), and the qubit retains partial coherence. This is standard in current quantum processors [6].

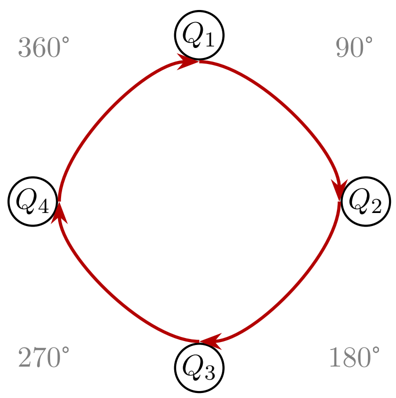
Proposition 2.2 (Egress record as physical signal)**.** *The egress record of a bilateral weak crossing is a physical signal — for a superconducting qubit, a microwave photon* *in the readout waveguide — that can be captured and routed to a neighbouring qubit to update its ingress potential. This propagation is causal (forward in τ), at the speed of light, and does not violate unitarity: the total state of qubit + field remains pure; only the qubit's reduced state changes.*

This is the mechanism of quantum feed-forward, well-established in quantum optics and superconducting circuits [9]. The bilateral framework provides a structural account of *why* this mechanism exists: the egress record of one crossing is structurally incorporated into the ingress potential of subsequent crossings via the bilateral mesh (Axiom A1: existence is relational; every state is defined by its intersections).

3 The Möbius Cascade Architecture

3.1 Setup

Consider N superconducting qubits arranged in a ring. Each qubit i is connected to qubit $i+1 \pmod N$ via a tunable coupler. The coupling includes a fixed 90° phase shift — implemented by a delay line or a parametric coupler — at each step. This is the half-Möbius topology: the electronic phase of the coupling signal accumulates 90° per step, exactly as in the $C_{13}Cl_2$ molecule [4].



Phase accumulated per step: 90°. *Full loop:* 360°. *Two loops:* 720°.

Figure 1: The Möbius cascade ring ($N = 4$ qubits). Each coupling arrow carries a 90° phase twist. After one full traversal the signal has accumulated 360°; after two traversals it returns to its original phase with a sign change (720°, spinor closure). Qubit indices are mod 4.

3.2 The Cascade Mechanism

The cascade is initiated by a single weak measurement on qubit Q_1:

1. Qubit Q_1 undergoes a weak crossing at angle $\theta < \pi/4$. An egress photon is emitted with amplitude $\sin\theta$; the qubit retains superposition with amplitude $\cos\theta$.

2. The photon is routed — via the tunable coupler and the 90° phase shifter — to Q_2, where it updates the ingress potential of Q_2 (a conditional displacement in phase space).
3. Q_2 undergoes its own weak crossing. An egress photon is emitted; residual superposition is retained; the signal propagates to Q_3.
4. After $N = 4$ steps, the signal has traversed the ring once, accumulating $4 \times 90^\circ = 360^\circ$ of phase. It arrives at Q_1 with the opposite sign ($e^{i\pi} = -1$) relative to the starting state.
5. Because the signal has the opposite sign, it *cannot* settle: it shifts the ingress potential of Q_1 in the opposite direction from where it started, driving the system into the next half-cycle. After a second traversal (720° total), the signal returns to Q_1 with the original sign — but by then a new cascade cycle has already begun.

The result is a self-sustaining oscillation: the cascade propagates around the ring continuously, each qubit alternately receiving a signal (updating its ingress potential) and emitting a signal (its egress record). The system produces a continuous output stream — one egress photon per qubit per traversal — without ever requiring the full superposition to be preserved across all qubits simultaneously.

3.3 Why the Topology Sustains the Cascade

The key is the sign flip after one traversal. In a conventional ring (zero twist per step), the signal would complete each loop with the same sign and the cascade would settle into a fixed point — the ring would lock at one eigenstate and stop oscillating. The 90° twist prevents this: the signal always arrives at Q_1 with the opposite sign after one loop, which drives another half-cycle. This is the topological analogue of the spinor's 720° property: two loops are required for the system to return to its starting state. The cascade never settles because it is always in the middle of its second loop.

Remark 3.1 (Connection to the 720° spinor)**.** *Theorem 16.1 of [1] derives the two mass prefactors of each fermion sector from the two half-cycles of the 720° bilateral spinor crossing. The Möbius cascade is the computational realisation of this structure: the two loops of the cascade correspond to the two half-cycles of the spinor, with the sign flip at the end of the first loop forcing the second half-cycle to begin. The cascade is a spinor implemented in superconducting hardware.*

4 Cascade Dynamics and Threshold Condition

4.1 Weak Crossing Amplitude

Let θ_i denote the crossing angle at qubit i. The egress amplitude at step i is $\sin\theta_i$ and the residual superposition amplitude is $\cos\theta_i$. For the cascade to self-sustain, the egress signal from qubit i must be sufficient to trigger a weak crossing at qubit $i+1$:

$$\sin\theta_i > \theta_{\min},$$

where $\theta_{\min}$ is the minimum crossing angle detectable by the readout circuit. For current superconducting processors, $\theta_{\min} \approx 0.05$–$0.1$ radians [6].

4.2 The Threshold at $\theta_c = \pi/4$

The bilateral mid-crossing occurs at $\theta_c = \pi/4$, corresponding to the facing direction $e^{i\pi/4}$ — the point at which egress and ingress amplitudes are equal ($\sin\theta_c = \cos\theta_c = 1/\sqrt{2}$). This is the threshold.

The residual superposition amplitude after k successive weak crossings at angle θ is $(\cos^2\theta)^k$. For the cascade to self-sustain, this residual must remain above the minimum detectable threshold $\theta_{\min}$ as $k \to \infty$:

$$\lim_{k\to\infty} (\cos^2\theta)^k > 0 \iff \cos^2\theta > 0 \iff \theta < \frac{\pi}{2}.$$

However, the feed-forward signal strength at step k scales as $(\sin\theta)(\cos^2\theta)^{k-1}$, which must also exceed $\theta_{\min}$ for the next crossing to be triggered. The cascade self-sustains when the residual exceeds the egress:

$$\cos^2\theta > \sin^2\theta \iff \cos 2\theta > 0 \iff \theta < \frac{\pi}{4}.$$

This gives the threshold exactly: $\theta_c = \pi/4$. Below this, residual superposition dominates; above it, the egress record dominates and the residual decays to zero within a finite number of steps.

- **Below threshold** ($\theta < \pi/4$): $\cos^2\theta > \sin^2\theta$. The residual superposition exceeds the egress record at each step. The cascade sustains itself; the residual superposition at each qubit provides the potential for the next weak crossing, while the egress record provides the feed-forward signal.

- **Above threshold** ($\theta > \pi/4$): $\sin^2\theta > \cos^2\theta$. Each crossing actualises more than it retains. The residual superposition decays geometrically to zero within a finite number of steps. The cascade terminates.

The transition at $\theta_c = \pi/4$ is sharp: the cascade persistence time scales as $-1/\ln(\cos^2\theta)$ steps, which diverges as $\theta \to \pi/4^-$ and drops abruptly to a finite value at $\theta = \pi/4^+$.

4.3 Cascade Coherence Time

The cascade coherence time T_{casc} is not limited by the individual qubit coherence time T_2. Each qubit is refreshed by the incoming feed-forward signal at a rate $f_{\text{casc}} = c_{\text{feed}}/N$, where c_{feed} is the feed-forward clock rate (set by the coupler bandwidth). As long as $f_{\text{casc}} \gg 1/T_2$, the cascade refreshes each qubit's potential before decoherence has time to destroy it. In current processors, $T_2 \sim 100$–$500\ \mu$s and tunable coupler bandwidths allow feed-forward at ~ 10–100 ns per step, giving a refresh rate 10^3–10^4 times faster than decoherence. The cascade coherence time is therefore set by the feed-forward fidelity and the residual superposition amplitude, not by T_2.

To see why the cascade coherence time greatly exceeds T_2: the cascade signal completes one traversal of N qubits in time $\Delta t = N/c_{\text{feed}}$. During this time, the residual superposition at each qubit decays by a factor $e^{-\Delta t/T_2}$. For the cascade to survive m traversals, the accumulated decay must remain above the cascade threshold:

$$(\cos^2\theta)^{mN} \cdot e^{-m\Delta t/T_2} > \sin^2\theta.$$

For $\theta \ll \pi/4$ and $\Delta t \ll T_2$, this gives a cascade lifetime $T_{\text{casc}} \approx T_2/(1-\cos^2\theta)^N \gg T_2$, diverging as $\theta \to 0$ (vanishingly weak measurement) and decreasing to T_2 as $\theta \to \pi/4$ (cascade threshold). For $\theta = \pi/8$ and $N = 4$: $T_{\text{casc}} \approx T_2/(1-\cos^2(\pi/8))^4 \approx 25\,T_2$.

5 Testable Predictions

The Möbius cascade makes four precise, falsifiable predictions:

1. **Output periodicity $2N$.** The sequence of egress outcomes (0 or 1 at each qubit, each traversal) has period $2N$ — not N, because two traversals are required for the 720° return. For $N = 4$: period = 8 events. This periodicity should be visible in the power spectrum of the output stream.

2. **Anti-correlation at separation N.** Output bits separated by exactly N steps (half a period) should be anti-correlated: if qubit Q_i registers outcome 0 at step t, then qubit Q_i should register outcome 1 at step $t + N$ (modulo noise). This is the direct signature of the 720° sign flip.

3. **Sharp phase transition at $\theta_c = \pi/4$.** As measurement strength is increased from zero, the output photon count rate should remain approximately constant until $\theta \approx \pi/4$, at which point it should drop abruptly to zero. The transition width should be much smaller than $\pi/4$ itself — a sharp rather than smooth crossover.

4. **Cascade coherence time $\gg T_2$.** The output stream should persist for a time significantly longer than the individual qubit T_2, with a persistence time scaling as $1/(1-\cos^2\theta)^N$ in the limit of low decoherence. For $\theta = \pi/8$ and $N = 4$: persistence $\sim T_2/(1-\cos^2(\pi/8))^4 \approx 25T_2$.

These predictions can be tested on current superconducting quantum processors with reconfigurable couplers and fast feed-forward (e.g., IBM Condor-class systems or equivalent).

6 Applications

6.1 Topological Quantum Random Number Generation

The output stream of the Möbius cascade is a sequence of weak measurement outcomes, each drawn from the Born rule applied to the current ingress state. Because the cascade never fully collapses, the output is not a repeated measurement of a fixed state but a continuous stream of correlated quantum random numbers. The prime-gap structure of the bilateral ladder [1] predicts that the correlation function of this stream should exhibit peaks at prime-gap separations — a signature distinguishable from classical pseudo-random sequences and from standard quantum random number generators.

6.2 Self-Clocked Quantum Processing

The cascade provides a built-in clock: one output photon per qubit per traversal, at a rate set by the coupler bandwidth. This eliminates the need for an external clock signal to synchronise gate operations. Computation can be performed by modulating the coupling strengths in synchrony with the cascade, effectively encoding gate operations as perturbations to the propagating signal.

6.3 A Note on Error Sensitivity

Because the cascade signal accumulates phase over multiple traversals, slowly varying phase errors at individual qubits will shift the output periodicity rather than terminating the cascade outright. A phase error of ϕ at qubit i shifts the signal phase by ϕ per traversal, accumulating 2ϕ over the full $720°$ return. Whether this amounts to useful error resilience depends on the specific noise model and is a question for detailed simulation rather than the present proposal. We flag it as a direction for further investigation, not a claim.

7 Implementation

A minimal proof-of-principle requires $N = 4$ superconducting qubits in a ring with:

- Tunable couplers capable of implementing a $90°$ phase shift per step (a delay line of length $\lambda/4$ at the operating frequency, or a parametric coupler with a controlled phase offset).
- Dispersive readout with adjustable measurement strength $\theta \in [0.05, \pi/2]$.
- Fast feed-forward: readout result captured and applied to the next qubit within ~ 100 ns (within current state-of-the-art [9]).
- Real-time output logging of all egress photons across all 4 qubits.

The cascade is initiated by a single weak measurement on Q_1 with $\theta \approx \pi/8$. The output stream is monitored for periodicity $2N = 8$, anti-correlation at separation $N = 4$, and persistence time. The measurement strength is then swept from $\pi/8$ to $\pi/2$ to observe the sharp phase transition at $\theta_c = \pi/4$.

IBM Condor-class processors (1000+ qubits with tunable couplers) and Rigetti Ankaa systems have the required capabilities. A 4-qubit sub-ring in a reconfigurable topology is straightforward to configure on either platform.

8 Relation to the Bilateral Framework

The Möbius cascade is a direct computational realisation of the bilateral framework's account of measurement [1]. Three connections are exact:

1. **Weak measurement as partial bilateral crossing.** The bilateral crossing angle θ maps directly to measurement strength. The egress record is the physical photon; the residual superposition is the ingress potential that sustains the cascade.

2. **Half-Möbius topology as the unit bilateral crossing i.** The 90° phase twist per step is the bilateral unit crossing $i = e^{i\pi/2}$, identified in Proposition 4.1 of [1] as the minimal step from the egress face to the ingress face. The cascade ring implements this step physically, at the gate level, in a superconducting circuit.

3. **720° return as spinor closure.** The bilateral framework derives the spinor property (Theorem 8.1 of [1]) from the closure condition of the bilateral crossing: two half-cycles of the 720° traversal are required for the crossing to return to its original state. The cascade ring implements this as a hardware constraint: two traversals are topologically required before the signal can return in phase. The anti-correlation at separation N (Prediction 2) is the experimental signature of this spinor closure in a computational setting.

The framework does not derive the specific hardware implementation — that is engineering. What it provides is the structural reason why the topology works: the cascade is sustained because the bilateral crossing structure requires 720° for closure, and the half-Möbius coupling enforces this requirement at the circuit level.

9 Conclusion

The Möbius cascade is a quantum computational architecture in which weak measurement is not a liability but the engine of the computation. By coupling N qubits in a half-Möbius ring, the 720° spinor return condition is enforced at the circuit level, sustaining a propagating cascade of weak collapses that produces a continuous output stream over timescales significantly exceeding individual qubit coherence times.

The architecture is grounded in two experimental facts: the bilateral crossing structure of quantum measurement, and the experimentally demonstrated half-Möbius electronic topology of $C_{13}Cl_2$ [4]. The four testable predictions — output periodicity $2N$, anti-correlation at N, sharp phase transition at $\theta_c = \pi/4$, and cascade coherence time $\gg T_2$ — are accessible to current superconducting quantum processors.

The deeper implication is that topology is a computational resource independent of coherence. The Möbius cascade does not preserve coherence across all qubits; it uses the topological structure of the coupling to sustain a computation that no individual qubit's coherence time could support alone.

References

[1] D. Low, "Infinity Zero: A Universal Synthesis of the Past, Present and Future," preprint, ontologia.co.uk, April 2026. Submitted to *Foundations of Physics*.

[2] D. Low, "The Uniqueness of $S^3 \times \mathbb{CP}^2$: Three Axioms, One Geometry, No Choice," preprint, ontologia.co.uk, May 2026.

[3] D. Low, "The IBM Half-Möbius Molecule as a Physical Realisation of the Bilateral Crossing," preprint, ontologia.co.uk, May 2026.

[4] L. Gross et al. (IBM Research et al.), "A molecule with half-Möbius topology," *Science*, March 2026. DOI: 10.1126/science.aea3321.

[5] J. Preskill, "Quantum Computing in the NISQ Era and Beyond," *Quantum* **2** (2018) 79.

[6] P. Krantz et al., "A quantum engineer's guide to superconducting qubits," *Appl. Phys. Rev.* **6** (2019) 021318.

[7] Y. Aharonov, D. Z. Albert, L. Vaidman, "How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100," *Phys. Rev. Lett.* **60** (1988) 1351.

[8] K. W. Murch et al., "Observing single quantum trajectories of a superconducting quantum bit," *Nature* **502** (2013) 211–214.

[9] D. Ristè et al., "Feedback control of a solid-state qubit using high-fidelity projective measurement," *Phys. Rev. Lett.* **109** (2012) 240502.

[10] D. Low, Bilateral mesh verification suite, github.com/dunstanlow/bilateral-mesh, 2026.