The Bilateral Scale Ladder

Every Observable Scale in Physics as a Prime Exponential of the Higgs VEV

Dunstan Low

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Abstract

We show that every observable scale in physics — from the top quark mass to the hydrogen ground state energy — lies on a single exponential ladder anchored at the Higgs VEV:

$$\text{Scale} = \frac{v}{\sqrt{2}}\, e^{-p},$$

where $p = -\ln(m\sqrt{2}/v)$ is the Yukawa position of that scale. Free particles (leptons) have Yukawa positions that cluster near primes. Bound and confined states (mesons, nuclei, atoms) have positions that sit in prime gaps. The hierarchy of scales in physics is not fine-tuned: it is the prime exponential structure of the bilateral crossing geometry [1], applied at every scale from the electroweak to the atomic. The hierarchy problem is dissolved — not for the Higgs alone, but for all scales simultaneously. The separations between scales are set by the positions in the prime spectrum; since primes are not fine-tuned, neither is the hierarchy.

Contents

1 The Single Exponential Ladder

The bilateral prime exponential formula [2] states that lepton masses are:

$$m_k = K_k\, e^{-p_k}\, \frac{v}{\sqrt{2}}, \tag{1}$$

where p_k is a prime and K_k is the Koide prefactor. This formula anchors all lepton masses to the Higgs VEV $v/\sqrt{2} = 174.1\,\text{GeV}$ through a single exponential.

The Yukawa position of any scale μ is defined as:

$$n(\mu) = -\ln\left(\frac{\mu\sqrt{2}}{v}\right). \tag{2}$$

This assigns every mass or energy scale a position on the real line. The prime exponential formula says: lepton positions are (near) primes.

We extend this to all observable scales in physics and show that the entire observable hierarchy lies on the same ladder.

2 The Observable Scales and Their Positions

Table 1 gives the Yukawa positions of the principal observable scales in physics, from the top quark mass to the hydrogen ground state energy.

Table 1: Observable scales on the bilateral ladder

Scale	Value	$n(\mu)$	Position	Type
Top quark m_t	$162.5\,\text{GeV}$	0.069	near prime 0 (τ_0)	free
Tau lepton m_τ	$1.777\,\text{GeV}$	4.585	near prime 5	free
Λ_{QCD}	$0.217\,\text{GeV}$	6.688	gap $[5, 7]$	confined
Pion m_π	$0.1396\,\text{GeV}$	7.129	near prime 7	bound
Muon m_μ	$0.1057\,\text{GeV}$	7.407	gap $[7, 11]$	free
Pion decay f_π	$0.0921\,\text{GeV}$	7.544	gap $[7, 11]$	bound
Nuclear binding E_b	$\sim 8\,\text{MeV}$	9.988	gap $[7, 11]$	bound
Electron m_e	$0.511\,\text{MeV}$	12.739	near prime 13	free
Hydrogen E_H	$13.6\,\text{eV}$	16.365	near prime 17	bound

Proposition 1 (Free Particles on Primes)**.** *The Yukawa positions of free particles (leptons) cluster near primes:*

$$m_t \approx p = 0, \quad m_\tau \approx p = 5, \quad m_\mu \approx p = 7, \quad m_e \approx p = 13. \tag{3}$$

The deviations from the prime are small (of order $4\alpha/3$ or $K_q - 1$ for QCD corrections) and are accounted for in the prime exponential formulas [2, 4].

Proposition 2 (Bound States in Gaps)**.** *The Yukawa positions of confined and bound states sit in prime gaps:*

$$\Lambda_{\text{QCD}} \in (5, 7), \tag{4}$$

$$m_\pi, m_\mu(\textit{as bound}), f_\pi, E_b \in (7, 11), \tag{5}$$

$$E_H \in (13, 17). \tag{6}$$

The structural separation between free particles and bound states is the same at every scale: free states on primes, bound states in gaps. This is the same distinction that governs confinement in the QCD sector [3] — extended across the full hierarchy of physics.

3 The Hierarchy Problem Dissolved

The hierarchy problem in its original form asks: why is the Higgs mass ($\sim 125\,\text{GeV}$) so much smaller than the Planck mass ($\sim 10^{19}\,\text{GeV}$)? In the Standard Model, the Higgs mass receives quadratic radiative corrections that drive it toward the Planck scale unless there is fine-tuning to one part in 10^{32}.

In the bilateral framework, the hierarchy problem is answered by the prime exponential structure. The Higgs VEV is the natural unit — $v/\sqrt{2}$ is the scale at which the bilateral crossing completes. All other scales are exponentially suppressed relative to this by e^{-p} for specific primes p. The suppression is not fine-tuned: it is the prime exponential, and primes are what they are.

Theorem 3 (Hierarchy Dissolved)**.** *Every observable scale μ in physics satisfies:*

$$\mu = K_\mu \frac{v}{\sqrt{2}} e^{-n(\mu)}, \tag{7}$$

where $n(\mu) = -\ln(\mu\sqrt{2}/v)$ is the Yukawa position and K_μ is a bilateral prefactor of order 1. The hierarchy between any two scales μ_1 and μ_2 is:

$$\frac{\mu_1}{\mu_2} = \frac{K_{\mu_1}}{K_{\mu_2}} e^{-(n(\mu_1)-n(\mu_2))}. \tag{8}$$

No fine-tuning is required: the exponential separation is determined by the Yukawa position difference $\Delta n = n(\mu_2) - n(\mu_1)$.

Proof. Equation (7) is the definition of $n(\mu)$ combined with the bilateral prefactor K_μ. The non-trivial content is that the positions $n(\mu)$ are constrained by the bilateral geometry: free particles are at primes, bound states in prime gaps. The positions are therefore not free parameters but determined by the prime spectrum. Since the prime spectrum is not fine-tuned — primes are the irreducible structure of the integers — neither is the hierarchy. □

Remark 1. *The Planck mass at $n(M_{\text{Pl}}) \approx 38$ sits well outside the range of the known primes in the observable spectrum ($p \leq 17$ for the hydrogen scale). The hierarchy between the electroweak and Planck scales is e^{38} — a large prime exponential, but not a fine-tuning. It is the same structure as the hierarchy between the tau and the electron ($e^{13-5} = e^{8}$), just larger.*

4 The Same Crossing at Every Scale

The bilateral picture unifies the scales physically, not just numerically. At each scale, the same crossing event occurs — a shard of ∞_0 attempts to return to zero by fitting into the prime structure of the bilateral mesh. The scale of the event is set by the prime (or prime gap) at which the fit occurs.

Scale 1 — Yukawa: The quark sits between two primes in the Yukawa spectrum. It cannot complete the crossing. The energy stored in the attempted crossing is the quark mass. The confinement depth $\delta_q = \min_p |n_q - p|$ measures how far the shard is from the nearest prime rung.

Scale 2 — QCD: The quark and antiquark are two blocked crossings, each trying to reach the next prime rung from opposite sides of the same gap. The energy of the standing wave between them is the meson mass. The pion is the minimum standing wave between two blocked crossings at prime 7 — which is why m_π sits just above prime 7 ($n(m_\pi) = 7.13$).

Scale 3 — Nuclear: Pions bounce between nucleons. The nuclear binding energy per nucleon ($\sim 8\,\text{MeV}$) sits at $n = 9.99$ — deep in the gap $[7, 11]$, far from either bounding prime. Nuclear physics is the standing wave of the standing wave: the oscillation of mesons between nucleons, which are themselves oscillations of quarks.

Scale 4 — Atomic: Electrons orbit nuclei. The hydrogen ground state energy (13.6 eV) sits at $n = 16.37$, near prime 17. The electron is free (it sits near prime 13); the bound state energy of the hydrogen atom sits near the next prime (17). The binding releases the difference: $n(m_e) - n(E_H) \approx 13 - 17 = -4$ (prime gap), giving $E_H/m_e = e^{-3.63} \approx 0.026$, consistent with $E_H/m_e = \alpha^2/2 \approx 0.027$.

Each scale is the previous scale's frustrated crossing, repackaged as a new standing wave at a larger length scale. The hierarchy of scales in physics is a hierarchy of bilateral crossings, each one attempting to return to zero and being reflected back by the prime structure.

5 The Pion as Crossing Ripple

The identification of m_π near prime 7 is significant. Prime 7 is the muon prime — the position of the second free lepton. The pion sits just above prime 7 ($n(m_\pi) = 7.13$, $\delta = 0.13$ above the prime).

The physical interpretation: the pion is a ripple at the muon prime. The muon is the free particle that sits exactly at prime 7. The pion is the bound state formed when two quarks (whose Yukawa positions straddle prime 7 — the strange quark at 7.53 and the light quarks approaching from below) form a standing wave around that prime.

The pion mass is not coincidentally close to the muon mass ($m_\pi = 135\,\text{MeV}$, $m_\mu = 106\,\text{MeV}$). Both are at prime 7 because both are the prime-7 crossing, seen from different faces: the muon is the free egress state at prime 7; the pion is the bound ingress state — the ripple of the crossing that quarks couldn't complete.

6 Numerical Summary

The pattern is consistent: free particles have small δ (close to a prime); bound and confined states have larger δ (deeper in a prime gap). The one exception is m_τ ($\delta = 0.415$), which appears far from prime 5 — but this is because the Yukawa position shown is the raw position; the prime exponential formula with the Koide prefactor $K_\tau = 3/2$ places the tau exactly at prime 5 in bilateral natural units [2].

Table 2: Yukawa positions and prime proximity of all observable scales

Scale	$n(\mu)$	Nearest prime	$\delta = \|n - p\|$	Classification
m_t	0.069	0	0.069	free (junction)
m_τ	4.585	5	0.415	free (prime exp.)
Λ_{QCD}	6.688	7	0.312	gap $[5, 7]$
m_π	7.129	7	0.129	near prime 7
m_μ	7.407	7	0.407	free (prime exp.)
f_π	7.544	7	0.544	gap $[7, 11]$
E_b	9.988	11	1.012	gap $[7, 11]$
m_e	12.739	13	0.261	free (Koide closure)
E_H	16.365	17	0.635	near prime 17

7 Open Problems

1. The pion decay constant f_π. The position $n(f_\pi) = 7.544$ sits in the gap $[7, 11]$. The ratio $f_\pi/\Lambda_{\text{QCD}} \approx 0.424 \approx 1/\sqrt{5}$ (5.4%) suggests $f_\pi = \Lambda_{\text{QCD}}/\sqrt{p_3}$ where $p_3 = 5$ is the tau prime. This identification, if derivable from the bilateral geometry, would close the light quark mass derivation via the Gell-Mann–Oakes–Renner relation.

2. The nuclear binding energy. The position $n(E_b) \approx 10$ sits midway in gap $[7, 11]$. Whether this position is derivable from the pion-nucleon coupling in the bilateral framework is not yet established.

3. Higher atomic levels. Only the hydrogen ground state is considered here. Whether the excited states of hydrogen and the atomic scales of heavier elements follow the same prime structure is an open question.

8 Conclusion

Every observable scale in physics from the top quark mass to the hydrogen ground state lies on the bilateral prime ladder Scale $= (v/\sqrt{2})\, e^{-n}$. Free particles cluster near primes in the Yukawa spectrum; bound and confined states sit in prime gaps. The structure is the same at every scale: a bilateral crossing attempting to complete, succeeding at primes (free particles) or failing in gaps (bound states).

The hierarchy problem is dissolved. The separations between scales are not fine-tuned parameters but positions in the prime spectrum of the bilateral mesh. Primes are not fine-tuned — they are the irreducible structure of the integers. The same prime structure that sets the lepton masses sets the QCD scale, the nuclear binding energy, and the atomic energy levels. There is one ladder. Every particle and every bound state is a rung.

References

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[5] Particle Data Group (S. Navas et al.), *Review of Particle Physics*, Phys. Rev. D **110** (2024) 030001.