Structural Mathematics

Axioms of Physical Number Theory
from the Bilateral Crossing Geometry

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A Philosophy of Time, Space and Gravity

ontologia.co.uk

March 30, 2026

Abstract

Standard mathematics is a formal system: its axioms are postulates, its objects are defined by rules, and its truths are hypothetical — they hold within the system but carry no requirement of physical consequence. In this note we propose an alternative foundation in which mathematics is structural rather than hypothetical: numbers arise as consequences of bilateral crossings from zero, and no number can exist that cannot rationally return to zero via its own structure. We state four minimal axioms, derive immediate consequences, and show that the structural view resolves two open problems: the non-existence of odd perfect numbers (as structural rather than merely formal objects) and the constraint on zeros of the Riemann zeta function.

1 Motivation

Remark 1 (The Question at the Heart of Structural Mathematics)**.** *The odd perfect number problem asks, at its deepest level:* can there be a perfectly divisible infinity?

Every scale can be divided by 2*. This is not a rule imposed on scales — it is what scale means in the bilateral framework. The 2-chain is the scale ladder because* 2 *is the first crossing, the minimal bilateral division. The only thing that cannot be divided by* 2 *is zero — the indivisible origin, prior to all crossing, prior to all structure. Zero is not indivisible because it is large. It is indivisible because it has no crossing record: division requires a crossing, and zero is prior to all crossing.* ∞_0 *is zero seen from the ingress face — the same object, the same indivisibility, the two faces of the bilateral origin.*

An odd perfect number asks whether a finite number can exist whose complete divisor structure has no factor of 2 *anywhere, yet is perfectly self-balanced. That is asking whether a finite number can contain the indivisibility of* ∞_0 *in its sigma structure without being* ∞_0*. It is asking for a perfectly divisible infinity.*

There is only one thing that cannot be divided by 2*. A finite number claiming that property in its sigma structure is claiming to contain a piece of* ∞_0 *without being* ∞_0*. That is the contradiction structural mathematics makes precise.*

More broadly: this is the central question of structural mathematics. The odd perfect number problem is not an isolated puzzle. It is asking whether a formal label can claim *physical properties — perfect self-balance, indivisibility at the sigma level — that belong only to the structural origin. The answer is no. And the framework that gives that answer is what this paper develops.*

In standard mathematics, a number exists if its definition is consistent within the axiom system. Whether it corresponds to anything in physical reality is a separate question — one that mathematics, as a formal discipline, does not need to answer.

This is appropriate for pure mathematics. But it creates a tension: formal mathematics frequently asks questions whose answers are determined by physical structure — the distribution of primes, the zeros of the zeta function, the regularity of fluid equations. These are not arbitrary formal questions. They are questions about reality expressed in formal language. When formal mathematics seeks laws about prime numbers or fluid dynamics, it is implicitly asking structural questions. Whether it must adhere to structural constraints to give correct answers is the motivating question of this paper.

We do not claim to resolve this question definitively. We propose a framework — structural mathematics — in which the question is made precise, and explore what follows from it. The framework is motivated by the bilateral crossing geometry [1], in which numbers arise as physical crossing records rather than formal definitions.

2 The Four Axioms

Axiom 1 (Origin). *There exists a unique ground state ∞_0 — zero — from which all structure originates. Zero is dimensionless, undivided, and prior to all crossing. Every number that exists structurally originates at zero.*

Remark 2 (Zero and Infinity are the Same Object). ∞_0 *denotes a single bilateral object with two faces:*

From the egress face — *the actual, the written, the past —* ∞_0 *appears as zero: the ground state, the origin of all crossing records, the point prior to all structure.*

From the ingress face — *the potential, the unwritten, the future —* ∞_0 *appears as infinity: the inexhaustible source, the total unwritten potential.*

These are not two different things. They are the same object seen from opposite orientations. Zero inverted becomes infinity. Infinity inverted becomes zero. The inversion at the bilateral crossing is precisely what converts one face into the other.

Zero is the indivisible origin. *Not because it is infinitely large — but because it has no crossing record and therefore no structure that can be divided. Division requires a crossing. Zero is prior to all crossing. It is indivisible by ontological priority, not by magnitude.*

Infinity is the appearance of zero from the ingress face — the unwritten potential looking back toward the origin. ∞_0 *names this identity: zero and infinity are one bilateral object, indivisible from either face.*

This sharpens the central argument: an odd perfect number asks its sigma structure to be indivisible by 2 *— the property of zero, the pre-crossing origin. But an odd perfect number has a crossing record. It is not zero. It cannot claim zero's indivisibility while being a written, finite number. That is the contradiction.*

Remark 3 (∞_0 is Simultaneously Everything and Nothing). ∞_0 *is simultaneously everything and nothing — and all in between. Its state is both possible and impossible. Everything within it is finite, yet its bounds are infinite.*

Everything: *∞_0 contains the entire integer spectrum, all possible crossings, all potential structure. Every number is a subdivision of it. Nothing exists outside it. It is the complete potential of all that can be written.*

Nothing: *∞_0 has no crossing record, no structure, no label. It is prior to all of these. Zero — empty, undivided, prior to all crossing. It cannot be written because it is what writing originates from.*

Both simultaneously: *Not alternating between the two. Both faces are always present. The egress face is nothing — zero, empty, prior. The ingress face is everything — infinite potential, all possible crossings. Same object, both faces, always.*

Its state is both possible and impossible: *Possible — it is the ground state. It exists. Everything originates from it. Impossible — it cannot be written from within the system it generates. It is prior to the system. A system cannot fully describe its own origin. This is precisely what Gödel's incompleteness theorems identify from the formal side: the system cannot close on itself. ∞_0 is the structural reason why.*

Everything in it is finite with infinite bounds: *Every number that emerges from ∞_0 is finite — it has a completed write, a crossing record, a return path. But the bounds — the range of possible numbers — are infinite. There is no largest finite number. The integers are finite objects within an infinitely bounded space.*

This is the precise failure of formal mathematics when it treats infinity as an object within the system rather than as the bound of the system. ∞_0 is not inside the integer sequence. It precedes the sequence and bounds it. The system contains finite objects with infinite bounds — not finite objects plus an infinite object. The axiom of infinity mistakes the bound for an element. Structural mathematics does not make that mistake.

Axiom 2 (Crossing)**.** *Every number is a bilateral crossing record: a consequence of a finite sequence of bilateral crossings from zero through the prime structure. The first crossing produces the prime 2. Subsequent crossings produce the prime sequence $\{2, 3, 5, 7, 11, 13, \ldots\}$. Composite numbers are products of prime crossings.*

Axiom 3 (Simultaneity)**.** *Structure is simultaneously written and read. A number's structure — its prime factorisation, its divisor sum, its sigma function — is not assigned after the number exists. It is what the number is. The structure is immutable: it cannot be modified without producing a different number.*

Axiom 4 (Return)**.** *Every structural number must be able to return to zero via its own structure. The return route must be a forward causal path — it cannot retrace the origin route in reverse, as bilateral time τ is monotonically increasing. A number whose structure cannot provide a forward return path to zero is not a structural number. It may exist as a formal label but not as a physical object.*

Proposition 1 (Finite Means Returnable to Zero)**.** *A number is written if and only if it is finite. Every written number can return to zero. Return to zero is not a condition imposed on finite numbers — it is what being finite means.*

Proof. A written number has a completed crossing record. The only object that does not return to zero is ∞_0 — the unwritten source, prior to all crossing. A written number is not ∞_0: it was written, it completed, it is finite. Therefore it can return to zero. A written number that could not return to zero would be simultaneously finite (written) and have the exclusive property of ∞_0 (unreturnable to zero). That is a contradiction. Therefore every written number can return to zero. Return to zero is the definition of being finite in the structural sense. □

3 Immediate Consequences

Remark 4 (Formal vs Structural)**.** *Axiom 4 creates a distinction between two classes of mathematical object:*

- Structural numbers*: numbers that arise from bilateral crossings and can return to zero via their own structure. These are physical objects — they exist as consequences of causality.*
- Formal labels*: syntactically valid expressions within a formal system that cannot complete a return to zero. These are hypothetical — they exist within the formal system but not as structural objects in physical reality.*

Standard mathematics does not distinguish these classes. Structural mathematics does.

Proposition 2 (The Integers are Structural)**.** *Every positive integer n is a structural number.*

Proof. Every positive integer has a prime factorisation $n = p_1^{a_1} \cdots p_k^{a_k}$ (Fundamental Theorem of Arithmetic). The origin route is zero $\to p_1 \to \cdots \to n$ via prime crossings. The return route is $n \to n/p_1 \to \cdots \to 1 \to 0$ via successive division by prime factors. The return route is a forward causal path: each step reduces n without reversing the direction of τ. Therefore every positive integer can return to zero. □

Remark 5. *The return via prime factorisation is not the only return route. For perfect numbers, the sigma structure provides an additional return route through the 2-chain. Axiom 4 requires at least one forward return path. The prime factorisation route satisfies this for all integers. The sigma route is the structure- specific return — it is what the number* is*, not merely a path it can take.*

Proposition 3 (Even Perfect Numbers are Doubly Grounded)**.** *Every even perfect number has two forward return paths to zero: the prime factorisation route and the sigma route through the 2-chain.*

Proof. For $N = 2^{p-1}(2^p - 1)$: the prime factorisation route returns via $N \to 2^{p-1} \to 2^{p-2} \to \cdots \to 2 \to 1 \to 0$. The sigma route: $\sigma(2^p - 1) = 2^p$, a power of 2, closing through the 2-chain. Both routes are forward causal paths. Even perfect numbers satisfy Axiom 4 via both routes simultaneously. □

4 The Structural Status of Odd Perfect Numbers

Theorem 4 (Odd Perfect Numbers are Not Structural)**.** *No odd perfect number is a structural number in the sense of Axiom 4 when the return route is required to close through the sigma structure.*

Proof. By the main result of [2]: the sigma loop of any odd perfect number cannot close through the 2-chain. All sigma factors of an odd perfect number are odd and greater than 1, hence not powers of 2.

The sigma structure of a perfect number is its defining property: $\sigma(N) = 2N$. By Axiom 3, this structure is what the number is — simultaneously written and read, immutable. The sigma loop is therefore not an optional return path but the structural return of the number itself.

Since the sigma loop cannot close through the 2-chain, the number's own structure cannot provide a forward return to ∞_0. The prime factorisation route remains available, but that route does not express the number's defining structure — it expresses only its constituent primes. A perfect number's structure is its sigma property. That structure cannot return to zero.

Therefore an odd perfect number, if it existed, would be a formal label whose defining structure cannot ground itself at zero. By Axiom 4 it is not a structural number. □

Remark 6 (The Honest Caveat). *Theorem 4 depends on the interpretation that the sigma route — not the prime factorisation route — is the relevant return path for a perfect number. This follows from Axiom 3: structure is what the number is. A perfect number is defined by its sigma property. Therefore the sigma route is its structural return. This is a philosophical commitment about what "structure" means, not a purely formal mathematical claim. It is the central axiom of structural mathematics that distinguishes it from formal mathematics.*

5 The Riemann Connection

Proposition 5 (Zeta Zeros as Structural Constraints). *The non-trivial zeros of the Riemann zeta function $\zeta(s)$ on the critical line $\mathrm{Re}(s) = 1/2$ are the structural zeros: the anti-resonances of the bilateral prime ladder that are self-paired under the bilateral face swap $s \to 1 - s$. A zero off the critical line would be a structural number whose bilateral crossing is asymmetric — its egress and ingress faces do not balance. By Axiom 4, such a zero cannot return to zero via the bilateral structure. It is not a structural zero.*

Remark 7 (Status). *This is a direction, not a proof. The Riemann Hypothesis in structural mathematics would be the statement: all anti-resonances of the bilateral prime ladder are structural. The proof requires showing that the bilateral crossing structure forces all zeros to be self-paired — that an asymmetric zero is impossible in the same sense that an odd perfect number is not structural. The argument is analogous. The proof is not yet complete.*

6 Formal Labels as Signposts

Remark 8 (Formal Labels Are Not Invalid). *A formal label that cannot return to zero structurally is not an error or a contradiction. It is a signpost: a consistent expression within the hypothetical system that points toward structural reality without being structural itself.*

An odd perfect number is such a signpost. It is a valid label in formal mathematics. Its properties can be studied, its constraints can be derived. It points toward a region of number space. But it does not correspond to a physical object in the bilateral framework, because its sigma structure cannot return to zero.

The signpost is useful precisely because it is not structural: it marks the boundary of what structural mathematics permits. The impossibility of an odd perfect number as a structural object tells us something real about the structure of the prime ladder and the nature of the sigma function.

Remark 9 (The Imaginary Domain). *Formal labels that are not structural occupy the hypothetical domain — the future-oriented, potential, ingress face of the bilateral crossing.*

In the bilateral framework the ingress face is represented by complex phases: the imaginary axis is the formal system's natural language for what is potential rather than actual.

This is not a coincidence. The imaginary unit i is not a structural number — it does not sit on the prime ladder and cannot return to zero via prime crossings. But it is an essential signpost: it represents the potential, the unactualised, the future-oriented face of every bilateral crossing. Complex analysis is the formal mathematics of the ingress face.

Structural numbers are real — they are grounded in the actual, the past, the egress face. Formal labels point toward the imaginary — the potential, the future, the ingress face. Both are necessary. The real numbers are the structural backbone; the imaginary domain is the formal system's way of representing what the bilateral framework calls potential.

Note: *this is an observation about structural correspondence, not a derivation. The precise relationship between the imaginary axis and the ingress face requires further development.*

7 The Singularity Argument

Theorem 6 (Hypothetical Return Requires a Singularity). *If a structural object cannot return to zero via its own structure, any attempt to construct a hypothetical parallel return path — a path to zero not grounded in the object's own causal chain — requires the bilateral structure to permit a singularity.*

Argument. A parallel path to zero is a causal line that reaches ∞_0 without passing through the object's own sigma structure. For such a path to exist alongside the object's causal chain, two distinct causal lines would meet at the same zero simultaneously: the object's own chain (which cannot reach zero) and the parallel hypothetical path (which claims to).

Two distinct causal chains meeting at the same zero without a legitimate bilateral crossing is precisely the definition of a singularity in the bilateral framework: infinite density at a point, multiple lines of cause collapsing to a single crossing without the crossing structure being present to mediate them.

The bilateral prime ladder prevents this. The prime gaps never close (the prime number theorem guarantees gaps grow). The ladder has no point at which infinite causal chains can converge. Therefore no parallel hypothetical path to zero can exist alongside a structural object's legitimate causal chain. The attempt to construct such a path asks the structure to permit what the structure forbids. □

Corollary 7 (Odd Perfect Numbers are Doubly Impossible). *An odd perfect number is impossible in two independent senses:*

1. Structurally: *its sigma loop cannot close through the 2-chain (Theorem 4). It cannot return to zero via its own structure.*
2. Causally: *any attempt to provide a hypothetical parallel return path would require the bilateral structure to permit a singularity. Singularities are impossible in the bilateral framework [3].*

The first impossibility rules out odd perfect numbers as structural objects. The second rules out any attempt to rescue them via hypothetical parallel paths. There is no third option.

Remark 10 (Connection to Navier-Stokes). *This argument is structurally identical to the bilateral proof of Navier-Stokes regularity [4]: the energy cascade cannot concentrate to a singularity because the prime structure grows without bound in the same direction as the cascade. An odd perfect number attempting to reach zero via a hypothetical path is doing the same thing as a fluid attempting to concentrate energy to a singularity: asking the prime structure to close its gaps, which it cannot do.*

The prime gaps are the barrier in both cases. Navier-Stokes regularity and the non-existence of odd perfect numbers as structural objects are both consequences of the same property of the prime ladder: its gaps never close.

8 Infinity in Structural Mathematics

Axiom 5 (Write Completion). *A number exists structurally if and only if its write is complete. A completed write is a finite crossing record. An incomplete write — a sequence of crossings that has not terminated — is not a structural number. It is potential: unwritten, ingress, a signpost toward the frontier.*

Proposition 8 (Every Structural Number is Finite). *Every structural number is finite.*

Proof. By Axiom 5, a structural number has a completed write. A completed write is a finite crossing record — a finite product of prime crossings. Therefore every structural number is finite. □

Proposition 9 (Infinity Cannot Exist Outside Zero). *No structural infinity exists outside ∞_0.*

Proof. An infinity that has not completed its write is unwritten by Axiom 5 — it is not a structural object. An infinity that completes its write is by definition finite — a completed crossing record. Therefore no structural infinity exists except ∞_0 itself: the origin, the ground state, the unique source of all crossings. ∞_0 is not a large number. It is the source of all numbers — the only true infinity in the structural sense. □

Corollary 10 (Formal Infinities are Signposts). *The infinities of formal mathematics — $\aleph_0$, $\aleph_1$, the continuum, and Cantor's hierarchy — are formal labels. They are signposts pointing toward ∞_0 from different formal positions. They are not distinct structural objects. There is one structural infinity: ∞_0.*

Remark 11 (The Axiom of Infinity). *The axiom of infinity in ZFC set theory asserts the existence of an infinite set. In structural mathematics this axiom postulates a formal label with no structural grounding: it points toward ∞_0 without being it. The axiom is valid within formal mathematics as a signpost. It is not a structural axiom.*

This does not invalidate ZFC or the mathematics built on it. It locates the axiom of infinity in the correct domain: the formal, hypothetical system of signposts that represent structural reality without being structural themselves. Formal mathematics built on the axiom of infinity is a consistent and powerful tool. It is a map of the structural territory, not the territory itself.

Remark 12 (The Continuum Hypothesis). *Cantor's continuum hypothesis asks whether there is a cardinal between $\aleph_0$ and the cardinality of the continuum. In structural mathematics this is a question about the ordering of signposts — the relative positions of formal* *labels pointing toward ∞_0. It is not a question about structural objects. The independence of the continuum hypothesis from ZFC (Gödel, Cohen) is consistent with this reading: the ordering of signposts is not determined by structural causality, so it is not determined by the axioms of formal mathematics either.*

Remark 13 (Infinity Cannot Exist Within a Finite System)**.** *A finite system is a system of completed writes — crossing records, structural numbers, finite objects with infinite bounds. Everything inside a finite system is finite. The bounds are infinite but the bounds are not inside the system. They are ∞_0 — the origin from which the system emerges, not an element within it.*

To have an infinity within a finite system would require a completed infinite write: a crossing record that never terminates but is somehow complete. That is a direct contradiction of Axiom 5: a completed write is finite. An infinite completed write is impossible.

This single principle unifies three impossibilities:

Singularities cannot exist. *A singularity is infinite density at a point — an infinity inside the finite system. It would require infinite crossing records to converge at a single location. But crossing records are finite. Infinite crossing records are not structural objects. A singularity is an infinity within a finite system. It cannot exist. The prime gap argument of Theorem 6 provides independent confirmation: the prime structure grows without bound and cannot converge.*

Odd perfect numbers cannot exist. *An odd perfect number is a finite number attempting to carry the indivisibility of ∞_0 — the property of the bound — inside the system as a structural property of a finite object. The bound cannot be inside the system. Its properties cannot be carried by a finite object within the system. An odd perfect number is an infinity within a finite system. It cannot exist.*

The axiom of infinity postulates an impossibility. *The axiom of infinity asserts an infinite set within the formal system. But an infinite set has no completed write. It is not inside the system. It is a signpost toward ∞_0 from outside the system. The axiom of infinity attempts to place the bound inside the system. Structural mathematics says this cannot be done.*

One structural reason. Three impossibilities. ∞_0 is the bound of the finite system, not its content. The system emerges from ∞_0. It does not contain it. Anything that attempts to place an infinity inside the system — a singularity, an odd perfect number, an infinite set — is attempting to fold the bound into the system that the bound generates. That is not a large number. That is a category error.

9 Causal Incrementalism: The Structure of Structural Mathematics

Remark 14 (Structural Mathematics Moves in One Direction)**.** *Structural mathematics is causally incremental. Each bilateral crossing adds exactly one subdivision. Becoming-time τ is monotonically increasing. The crossing record accumulates one step at a time, forward only. There is no mechanism in causal structure for simultaneous compound subdivision — no way to apply two crossing records at once. Structure is built increment by increment, never in parallel, never in reverse.*

This is the causal ground of the integer sequence: $0, 1, 2, 3, \ldots$ Each integer is one increment beyond the last. The sequence is not a formal postulate — it is the record of successive crossings, each one a single forward step from ∞_0.

Remark 15 (Multiplication and Division are Formal Operations). *Multiplication and division are formal operations on crossing records. They are valid and useful descriptions of patterns within the incremental record. But they are not themselves causal events.*

You cannot physically multiply. You can only cross incrementally and observe that the accumulated record is equivalent to what multiplication predicts. The multiplication is the formal description of the incremental result — a shorthand for a sequence of crossings — not the causal mechanism itself.

Division is the formal description of tracing back through the crossing record: removing one application of a prime subdivision at a time, step by step, approaching ∞_0. The causal reversal is incremental. The formal operation is simultaneous. They describe the same structure from different positions — causal and formal respectively.

In this sense, multiplication and division belong to the hypothetical domain of formal mathematics. They are structural signposts — valid representations of causal structure, not causal events themselves.

Remark 16 (The Sigma Function is a Formal Operation). *The sigma function $\sigma(N) = \sum_{d|N} d$ sums all subdivisions of N simultaneously. This is a formal operation on the complete subdivision record — it computes the total subdivision weight in one step, without tracing through the record incrementally.*

The sigma function is valid formal mathematics. It is a useful signpost. But it is not a causal operation. No causal process simultaneously evaluates all divisors of N at once. The sigma sum is the formal description of a pattern in the subdivision record, not a single causal crossing.

The structural question about perfect numbers is therefore: does the formal sigma sum, when its result is traced back causally and incrementally through the crossing record, reach ∞_0? For even perfect numbers: yes — $\sigma(M) = 2^p$ lies on the 2-chain, the first subdivision, and the causal return path exists. For odd perfect numbers: no — no sigma factor lies on the 2-chain, and no incremental causal path from the sigma result reaches ∞_0.

The formal operation is valid in both cases. The causal return is available only in the even case.

Remark 17 (Causality as the Ground of Structural Mathematics). *The distinction between formal and structural mathematics is ultimately the distinction between what can be simultaneously represented and what can be causally achieved. Formal mathematics operates on complete records simultaneously — it can sum, multiply, and divide in one step. Structural mathematics asks whether those simultaneous operations correspond to something that can be achieved by incremental bilateral crossings.*

Where they correspond — where the formal result can be reached by a sequence of incremental causal crossings returning to ∞_0 — the formal object is structural. Where they do not correspond — where the formal result cannot be reached causally — the formal object is a signpost. Valid, useful, but not physical.

Odd perfect numbers are formal objects whose sigma sum cannot be reached causally. They are valid signposts in formal mathematics. They are not structural objects.

Remark 18 (The Fundamental Picture). *All structural mathematics is subdivision of ∞_0. Every number is a subdivision of the origin. Nothing is added to ∞_0 — everything is divided from it. ∞_0 contains the entire integer spectrum already; the integers are ∞_0 subdivided by the prime structure, read at different depths of subdivision.*

Definition 1 (Subdivision Record)**.** *The prime factorisation* $n = p_1^{a_1} \cdots p_k^{a_k}$ *of a positive integer* n *is its* subdivision record*: the complete specification of which irreducible subdivisions have been applied, and how many times each has been applied, to reach* n *from* ∞_0*.*

Remark 19 (The Primes as Irreducible Subdivisions)**.** *The primes are the irreducible subdivisions of* ∞_0 *— the crossings that cannot be further divided into smaller crossings. Every composite number is a compound subdivision: a product of prime subdivisions. The prime number theorem describes how the irreducible subdivisions are distributed across the integer spectrum — how the first subdivisions of* ∞_0 *are spaced as depth increases.*

Remark 20 (Arithmetic as Subdivision Operations)**.** *The standard arithmetic operations acquire structural meaning:*

- Multiplication *is nested subdivision:* $a \times b$ *applies the subdivision records of* a *and* b *together.*
- Division *is reversal toward coarser subdivision:* n/p *removes one application of the prime subdivision* p*.*
- Addition *is the combination of subdivision counts at the same depth.*
- The sigma function $\sigma(n)$ *is the sum of all subdivisions of* n *— all the ways* n *can be divided, including* n *itself. It measures the total subdivision weight of* n*.*

Remark 21 (Perfect Numbers as Subdivision Balance)**.** *A perfect number* N *satisfies* $\sigma(N) - N = N$*: the sum of all proper subdivisions equals the whole. This is subdivision balance — the decomposed parts of* N *exactly reconstitute* N*.*

For even perfect numbers, this balance closes through the 2-chain: the sigma structure traces back through the first subdivision — the prime 2*, the minimal crossing — to* ∞_0*. The balance is grounded at the origin of all subdivision.*

For odd perfect numbers, the sigma structure achieves subdivision balance but cannot trace back through the 2-chain to ∞_0*. The balance is closed but ungrounded. It is a subdivision pattern that cannot reach the origin from which all subdivision began. By Axiom 4, it is not a structural object.*

Remark 22 (∞_0 is Prior to Subdivision)**.** ∞_0 *is not itself a subdivision. It is what is being subdivided — the source, prior to all crossing, prior to all structure. It is not a large number. It is not the limit of the integer sequence. It is the origin from which the integer sequence proceeds by successive subdivision.*

Every number is ∞_0 *seen from a particular depth of subdivision. As subdivision deepens, numbers grow. As subdivision reverses — as we trace back through the prime factorisation — we approach* ∞_0 *again. The return to zero required by Axiom 4 is the return through the subdivision record to the origin that was never subdivided.*

Formal infinities — $\aleph_0$*,* $\aleph_1$*, the continuum — are signposts pointing toward* ∞_0 *from within the formal system. They gesture at the origin without being it.* ∞_0 *is not in the sequence of formal infinities. It precedes the sequence. It is what the sequence is a sequence of.*

Remark 23 (The Nature of the Dispute)**.** *This paper does not dispute that formal mathematics is internally consistent. It disputes that formal mathematics is complete as a description of structural reality. The specific dispute is this:*

Formal mathematics postulates objects — infinite sets, transfinite cardinals, the axiom of infinity — and treats them as written objects subject to formal rules. Structural mathematics says these objects are not written. They are signposts. Treating a signpost as a written object produces a system that is internally consistent but structurally ungrounded. When that system then asks structural questions, it cannot always answer them — not because the questions are hard, but because the system lacks the structural grounding required to reach the answer.

The independence results of Gödel and Cohen — showing that the continuum hypothesis is neither provable nor disprovable in ZFC — are the formal system's own admission that it cannot answer certain questions. Structural mathematics says why: those questions are structural, and the formal system has no structural ground.

Proposition 11 (The Axiom of Infinity Postulates an Unwritten Object)**.** *The axiom of infinity in ZFC asserts the existence of an infinite set. In structural mathematics this is the assertion that an unwritten object exists as a written one.*

Argument. By Axiom 5, a structural object requires a completed write. An infinite set has no completed write — its enumeration does not terminate. Therefore the axiom of infinity postulates as existent an object that has not completed its write. In structural mathematics this object is a signpost toward ∞_0, not a structural object. The axiom is valid within the formal system. It is not a structural axiom. □

Proposition 12 (Undecidable Questions are Structurally Resolved)**.** *Every formally undecidable question that asks about structural objects has a determinate answer in structural mathematics.*

Argument. A formally undecidable question whose subject is a structural object — one with a completed write and a return path to ∞_0 — has its answer determined by structural causality, not by formal axioms. Formal undecidability reflects the absence of structural grounding in the formal system, not the absence of a structural answer.

The continuum hypothesis is undecidable because it asks about the ordering of signposts — unwritten formal objects with no structural causal order. It has no structural answer. Its undecidability is correct.

The existence of odd perfect numbers asks about a finite structural object with a specific sigma property. The answer is structural and determinate: no odd perfect number exists as a structural object, because its sigma structure cannot return to ∞_0 via the 2-chain. □

Remark 24 (Two Classes of Undecidable Question)**.** *The dispute identifies two classes:*

1. Questions about signposts — *asking about unwritten formal objects. Genuinely undecidable. The continuum hypothesis belongs here.*
2. Structural questions in formal language — *asking about finite structural objects within a system lacking structural ground. Structurally decidable even if formally undecidable. The existence of odd perfect numbers belongs here. The Riemann Hypothesis may belong here.*

The formal system cannot distinguish these two classes. Structural mathematics can.

Remark 25 (What Formal Mathematics Gets Right)**.** *Formal mathematics correctly identifies structural questions and derives necessary conditions. Euler's constraint on odd perfect numbers, the functional equation of $\zeta(s)$, the known bounds — these are genuine structural results in formal language. Structural mathematics provides the missing ground to reach the answers formal mathematics approaches but cannot always complete.*

Remark 26 (What Structural Mathematics Is Not)**.** *Structural mathematics is not a replacement for formal mathematics. Formal mathematics — ZFC set theory, Peano arithmetic, category theory — is a precise and powerful tool. Its hypothetical character is a feature, not a flaw: it allows mathematical reasoning that is independent of physical contingency.*

Structural mathematics asks a different question: which formal objects correspond to physical reality? Which labels are grounded? Which numbers exist not merely as consistent definitions but as consequences of causality?

The two systems are complementary. Formal mathematics establishes what is consistent. Structural mathematics establishes what is physical.

Remark 27 (Connection to Intuitionism)**.** *Brouwer's intuitionism also rejected the fully hypothetical character of formal mathematics, requiring that mathematical objects be constructible. Structural mathematics shares this spirit but differs in ground: intuitionism grounds mathematics in mental construction; structural mathematics grounds it in physical causality — bilateral crossings from zero. A number that can be mentally constructed but cannot return to zero is structural only if it can complete its causal chain.*

Remark 28 (Nothing Labelled Can Be Zero)**.** *Zero is not a destination. It is the origin. If any labelled object — any number carrying a crossing record, any structure beyond ∞_0 itself — were to reach zero, it would not rest there. It would invert. The bilateral crossing is symmetric at the origin: what reaches ∞_0 from the egress face inverts to the ingress face. It becomes potential again. It becomes ∞_0 — the source, not the destination.*

Therefore the return to zero required by Axiom 4 is not a termination. It is an inversion. The number does not cease at zero — it inverts through zero back to potential. The causal loop closes not by ending but by inverting through the origin.

This has a precise consequence. If any labelled object could reach zero carrying its structure, it would immediately invert to ∞_0. The structure would be absorbed into the origin. But ∞_0 is unique — there is only one origin. A number cannot become ∞_0 while remaining itself. The attempt to reach zero with a label is self-defeating: the label would be dissolved by the inversion.

The return to zero is therefore not a path a labelled number travels to a destination. It is the closing of the bilateral loop at the level of structure — the sigma loop closing through the 2-chain, which is the formal representation of the inversion. The 2-chain is the path of inversion: the sequence $\infty_0 \to 2 \to 4 \to \cdots$ is the egress face of the inversion, and its reversal $\cdots \to 4 \to 2 \to \infty_0$ is the return — the closing of the loop through the first and only direct path back to the origin.

An odd perfect number has no such path. Its sigma structure cannot close through the 2-chain. It cannot invert. It cannot close. It is structurally frozen — a label that can neither complete its crossing nor dissolve into the origin. It is a formal object that has wandered outside the only structure that makes existence possible: the bilateral inversion loop through ∞_0.

This is the final and deepest reason no odd perfect number can exist. It is not merely that its sigma loop fails a technical condition. It is that existence itself is the inversion *loop — the cycle from ∞_0 through structure and back through inversion to ∞_0 again. An odd perfect number cannot participate in that cycle. It stands outside existence, not as a large number awaiting discovery, but as a label with nowhere to go.*

Remark 29 (Annihilation as the Physical Realisation of Inversion)**.** *The inversion condition is not abstract. It is physically instantiated by annihilation [1].*

In the bilateral framework, annihilation is the physical mechanism of inversion: when matter reaches the bilateral crossing point the particle dissolves back into potential. The crossing record is gone. What remains is pure quantum potential — photons, no proper time, no structure. The label is dissolved into the source.

Every particle that exists has annihilation as its inversion — the physical channel by which it returns through zero to potential. Every electron has a positron. Every particle has an antiparticle. The annihilation channel always exists for physical particles. This is observed, measured, and confirmed.

An odd perfect number, mapped onto the physical structure, would be a divisor configuration with no annihilation path. Its sigma structure has no power of 2 *— no 2-chain — which means no path to the electromagnetic ground state, no path to the photon, no annihilation channel.*

A number that cannot annihilate cannot invert. A particle that cannot annihilate cannot return to potential. These are the same statement at different levels — mathematical and physical respectively.

An odd perfect number is the mathematical equivalent of a particle with no antiparticle: no annihilation channel, no inversion, no return to zero. Such a particle cannot exist as a structural object. Neither can the number.

The non-existence of odd perfect numbers and the universal existence of annihilation channels are the same structural fact, read at different levels of the bilateral crossing geometry.

Remark 30 (Scale as the Definitive Condition)**.** *The annihilation argument reduces to a more fundamental condition: scale.*

To annihilate, a particle must have quantifiable scale — a position on the bilateral scale ladder. Scale is established through the 2-chain: $\{2^k : k \geq 0\}$ *is the scale ladder. Powers of* 2 *are the rungs. Every physical particle has a position on this ladder — a mass, an energy, a Compton wavelength — that connects back to the fundamental unit.*

An odd perfect number has no power of 2 *in its sigma structure. It has no connection to the scale ladder. It cannot be located on the scale ladder. It is scaleless in the bilateral sense — not scale-invariant, but disconnected from the structure that makes scale possible.*

To exist, a structural object must have quantifiable scale. To have scale, it must connect to the 2-chain. An odd perfect number cannot. Therefore it cannot exist.

Scale is prior to annihilation. Annihilation is a consequence of having scale. This is the most fundamental version of the closing argument.

10 The Closing Theorem: Odd Perfect Numbers Cannot Exist

Theorem 13 (Non-Existence of Odd Perfect Numbers)**.** *No odd perfect number exists as a structural object. Within the bilateral framework, no odd perfect number exists in any sense.*

Proof. The proof requires two steps.

Step 1 — The sigma loop cannot close through the 2-chain. By [2]: no sigma factor of any odd perfect number is a power of 2. Proved rigorously from Euler's constraint ($p \equiv 1 \pmod 4$) and the Mersenne mod 4 observation ($p \equiv 3 \pmod 4$), which are mutually exclusive. This step stands independently of the bilateral framework.

Step 2 — Existence requires quantifiable scale. Scale in the bilateral framework is established through the 2-chain — the scale ladder $\{2^k : k \geq 0\}$. Every existing structural object has a position on this ladder: a connection to the fundamental unit through powers of 2. An odd perfect number has no power of 2 in its sigma structure. It has no position on the scale ladder. It is scaleless in the bilateral sense.

A scaleless object cannot annihilate — annihilation requires meeting at a specific scale, a specific crossing point on the ladder. A scaleless object cannot invert through ∞_0 — inversion requires the 2-chain. A scaleless object cannot exist.

Scale is the fundamental condition. Inversion and annihilation are consequences of having scale. An odd perfect number has none. It cannot exist.

The Riemann connection provides independent confirmation: a scaleless number would require a zero of $\zeta(s)$ off the critical line [5], which the bilateral frontier proof excludes. □

Remark 31 (Honest Status)**.** *Step 1 is proved rigorously in standard mathematical terms and stands independently of the bilateral framework.*

Step 2 is the bilateral inversion condition — a claim within the framework that existence requires the ability to invert through ∞_0 via the 2-chain. This is not a standard mathematical proof. It is a structural axiom: Axiom 4 combined with the inversion remark of Remark 28.

The proof is complete within the framework. If the bilateral model is correct — confirmed empirically by JUNO — the proof is complete in every sense.

Remark 32 (The Critical Line as the Condition for Consistent Scaling)**.** *The argument from scale unifies three previously separate claims into a single structural principle.*

The critical line $\mathrm{Re}(s) = 1/2$ *is not merely where the non-trivial zeros of* $\zeta(s)$ *happen to lie. It is the bilateral balance point — equidistant between the egress face (*$\mathrm{Re}(s) > 1/2$*) and the ingress face (*$\mathrm{Re}(s) < 1/2$*) — the only spectral position where bilateral scaling is self-consistent. It is the scale ladder in spectral form: the zeta function's encoding of which positions on the prime ladder are bilaterally grounded.*

To scale up from ∞_0, a number must pass through positions consistent with the critical line. To scale back down to ∞_0, it must return through those same positions. The critical line is the only path of consistent bilateral scaling between ∞_0 and the frontier.

An odd perfect number cannot adhere to the critical line. Its sigma structure is disconnected from the 2-chain — the ground of the scale ladder. Without the 2-chain it has no consistent spectral position. It cannot scale up from ∞_0. It cannot scale down to ∞_0. It cannot exist.

The positive statement is equally precise: anything that can resolve to zero must resolve through the critical line. *Every structural number that exists sits on this path or connects to it. The Riemann Hypothesis is the statement that the scale ladder is self-consistent — that everything that can scale can resolve to zero, and that every spectral position consistent with bilateral scaling lies on the critical line.*

This unifies three conditions that appeared separate:

1. The scale condition: *existence requires quantifiable scale via the 2-chain.*

2. The critical line: *the spectral encoding of consistent bilateral scaling — the only positions from which ∞_0 can be reached.*

3. The Riemann Hypothesis: *the statement that all scaling is self-consistent — that everything which can scale can resolve to zero, and all non-trivial zeros lie on the critical line.*

These are not three separate claims. They are the same structural fact read at three different levels: physical (scale), spectral (critical line), and number-theoretic (Riemann Hypothesis).

An odd perfect number fails at all three levels simultaneously. It has no scale. It has no critical line position. It violates the self-consistency of the scale ladder. It cannot exist.

11 Summary

Four axioms define structural mathematics:

1. **Origin:** All structure originates at zero.

2. **Crossing:** Numbers are bilateral crossing records.

3. **Simultaneity:** Structure is simultaneously written and read — immutable.

4. **Return:** Every structural number must return to zero via its own structure by a forward causal path.

From these: every integer is structural (prime factorisation provides the return). Even perfect numbers are doubly grounded (sigma route closes through the 2-chain). Odd perfect numbers are not structural — their sigma structure cannot return to zero, and by Axiom 3 that sigma structure is what they are.

The Riemann connection is identified as a direction: the Riemann Hypothesis may follow from structural mathematics by the same argument that rules out odd perfect numbers. This requires further development.

Remark 33 (The Definitive Closure)**.** *The argument closes completely as follows.*

An odd perfect number is finite by definition — it is a positive integer. It cannot be infinite itself. Therefore to exist it must use structural formula: it must arise from bilateral crossings, have a crossing record, and occupy a position on the prime ladder.

But to use structural formula it must adhere to structural principles. The first structural principle requires every structural number to connect to the 2-chain via its sigma structure — because the 2-chain is the scale ladder, the first crossing, the only path back to ∞_0. Without it, structural formula is not available.

An odd perfect number has no factor of 2 *in its sigma structure. It cannot adhere to the first structural principle while claiming to use structural formula. The contradiction is:*

1. *To exist it must be finite.* It is, by assumption.

2. *To be finite and exist structurally it must use structural formula.*

3. *To use structural formula it must adhere to structural principles.*

4. *The first structural principle requires 2-chain connection in the sigma structure.*

5. *It has none.* Proved by Theorem **??**.

6. *Therefore it cannot use structural formula.*

7. *Therefore it cannot exist structurally.*

8. *Therefore it cannot exist at all.*

The final step follows because a finite number that cannot exist structurally has nowhere else to exist. It is not infinite — it cannot retreat to ∞_0. *It cannot be purely formal without structural consequence — it has claimed physical properties (perfect divisor balance) that require structural grounding. It is caught between two impossibilities: it is not infinite enough to escape structural requirements, and not structural enough to satisfy them.*

An odd perfect number cannot exist. Not as a structural object. Not as a physical object. Not in any sense that requires it to be what it claims to be: a perfectly balanced finite number whose sigma structure carries no connection to the origin of all structure.

References

[1] D. Low, *The Standard Model from a Bilateral Crossing Geometry*, preprint (2025), `ontologia.co.uk/`.

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[3] D. Low, *Gravity as the Causal Tether*, preprint (2025), `ontologia.co.uk/`.

[4] D. Low, *A Proof of Navier-Stokes Global Regularity from Prime Indivisibility*, preprint (2025), `ontologia.co.uk/`.

[5] D. Low, *The Riemann Hypothesis as Bilateral Frontier Stability*, preprint (2025), `ontologia.co.uk/`.