The Top Quark Mass from Bilateral Asymmetry

$m_t = (v/\sqrt{2})\, \exp\big(-(K_{\text{up}} - K_{\text{down}})\big)$

The Junction of the Colour Ladders

Dunstan Low

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Abstract

We derive the top quark mass from the bilateral asymmetry between the up-type and down-type quark Koide values. The top quark sits at τ_0 — the crossing point itself, the base of all bilateral ladders simultaneously. Its Yukawa position in the MS-bar scheme is $\delta_t = K_{\text{up}} - K_{\text{down}} = 4/(3\varphi) - 3/4 = (8\sqrt{5} - 17)/12$, where φ is the golden ratio [2]. This gives:

$$m_t(\overline{\text{MS}}) = \frac{v}{\sqrt{2}} \exp\left(-\frac{8\sqrt{5} - 17}{12}\right) = 161.68\,\text{GeV},$$

against the observed $162.5\,\text{GeV}$ (0.51%). The top quark does not sit on a prime and is not derived from a prime exponential like the leptons. Instead it sits at the junction of the up-type and down-type colour ladders, at the position that encodes the asymmetry between them. The physical picture: the top quark is a shard of ∞_0 that has not yet committed to either colour ladder. It sits so close to the crossing point that it decays via the weak interaction before colour confinement can claim it.

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1 The Top Quark as Junction State

In the bilateral framework [1], the Yukawa spectrum maps fermion masses to positions on the real line via $n_f = -\ln Y_f = -\ln(m_f\sqrt{2}/v)$. Free particles (leptons) sit on primes; confined particles (quarks) sit in prime gaps [3]. The Koide prefactor K_q gives the fit fraction — how much of the $S^3 \times \mathbb{CP}^2$ crossing geometry the shard closes at that position.

The up-type and down-type quark sectors have distinct Koide values [2]:

$$K_{\text{down}} = \frac{3}{4} = \frac{\dim_{\mathbb{R}}(S^3)}{\dim_{\mathbb{R}}(\mathbb{CP}^2)}, \qquad K_{\text{up}} = \frac{4}{3\varphi}, \tag{1}$$

satisfying $K_{\text{up}} \times K_{\text{down}} = 1/\varphi$ and $K_{\text{eg}} \times K_{\text{down}} = K_\nu = 1/2$.

The up-type quarks use primes $\{0, 5, 11\}$; the down-type use $\{3, 7, 11\}$. The top quark is assigned prime $p = 0$ — the crossing point τ_0 itself.

Definition 1 (Junction State). *A* junction state *is a particle whose Yukawa position is $p = 0$ — the bilateral crossing point τ_0. At τ_0 the S^3 and $\mathbb{CP}^2$ geometries are not yet separated. The junction state sits at the base of both the electroweak and colour ladders simultaneously.*

The top quark is the unique junction state. Its Yukawa coupling $y_t \approx 1$ reflects its proximity to τ_0: no prime exponential suppression has occurred because no prime has been passed.

2 The Bilateral Asymmetry as Yukawa Position

If the top quark sits at τ_0, its Yukawa position is not exactly zero — it is displaced from τ_0 by the bilateral asymmetry between the two quark sectors.

Theorem 1 (Top Yukawa Position). *The top quark Yukawa position in the MS-bar scheme is:*

$$\delta_t = K_{\text{up}} - K_{\text{down}} = \frac{4}{3\varphi} - \frac{3}{4} = \frac{8\sqrt{5} - 17}{12}. \tag{2}$$

Proof. At τ_0, the bilateral geometry has two faces: the up-type face (Koide value K_{up}) and the down-type face (Koide value K_{down}). The top quark is the up-type junction state. Its displacement from τ_0 is set by the asymmetry between the two faces: $\delta_t = K_{\text{up}} - K_{\text{down}}$. The algebraic simplification:

$$K_{\text{up}} - K_{\text{down}} = \frac{4}{3\varphi} - \frac{3}{4} = \frac{16 - 9\varphi}{12\varphi}. \tag{3}$$

Since $\varphi = (1 + \sqrt{5})/2$, we have $9\varphi = (9 + 9\sqrt{5})/2$ and $12\varphi = 6(1 + \sqrt{5})$, giving:

$$\frac{16 - 9\varphi}{12\varphi} = \frac{8\sqrt{5} - 17}{12}. \tag{4}$$

□

The exact form $(8\sqrt{5} - 17)/12$ has a transparent geometric reading:

- $8 = N_c \times \dim_{\mathbb{R}}(S^3) - 1 = 3 \times 3 - 1$: colour charge times electroweak dimension, minus the singlet.
- $17 = p_7$: the seventh prime.
- $12 = N_c \times \dim_{\mathbb{R}}(\mathbb{CP}^2) = 3 \times 4$: colour charge times colour space dimension.

3 The Top Quark Mass

Theorem 2 (Top Quark Mass)**.** *The top quark mass in the* $\overline{\text{MS}}$ *scheme is:*

$$m_t(\overline{\text{MS}}) = \frac{v}{\sqrt{2}}\, e^{-\delta_t} = \frac{v}{\sqrt{2}}\, \exp\left(-\frac{8\sqrt{5}-17}{12}\right). \tag{5}$$

Proof. The Yukawa coupling at the top mass scale satisfies $Y_t(m_t) = e^{-\delta_t}$ (from the bilateral Yukawa position δ_t with $\gamma = 1$ [5]). The mass relation $m_t = Y_t \times v/\sqrt{2}$ gives the result. ☐

Table 1: Top quark mass prediction vs. observation [6]

Quantity	Value	
v	246.22 GeV	(input)
$\varphi = (1+\sqrt{5})/2$	1.6180...	(bilateral fixed point)
$\delta_t = (8\sqrt{5}-17)/12$	0.07404	(predicted)
$m_t(\overline{\text{MS}})$ predicted	161.68 GeV	
$m_t(\overline{\text{MS}})$ observed	162.5 GeV	
Deviation	0.51%	
m_t^{pole} (with 1-loop QCD)	see §4	
m_t^{pole} observed	172.76 GeV	

4 The Same Structure as the Tau

The tau lepton mass formula [4] is:

$$m_\tau = \frac{3}{2}\, \exp\left(-5 + \frac{4\alpha}{3}\right) \frac{v}{\sqrt{2}}, \tag{6}$$

where $4\alpha/3$ is the one-loop QED correction. The correction shifts the effective exponent *upward* (reduces the suppression), raising the tau mass slightly above the tree-level prediction.

The top quark formula has the same structure:

$$m_t(\overline{\text{MS}}) = \frac{v}{\sqrt{2}}\, e^{-\delta_t}, \qquad \delta_t = K_{\text{up}} - K_{\text{down}}. \tag{7}$$

The bilateral framework naturally gives masses in the $\overline{\text{MS}}$ scheme, not the pole scheme. The $\overline{\text{MS}}$–pole conversion for the top quark is:

$$m_t^{\text{pole}} = m_t(\overline{\text{MS}}) \left(1 + \frac{4\alpha_s(m_t)}{3\pi} + \ldots\right), \tag{8}$$

where $\alpha_s(m_t) \approx 0.108$, giving a correction of approximately +4.6%. This moves the prediction from 161.7 GeV to approximately 169.1 GeV, within 2.1% of the observed pole mass 172.76 GeV.

The residual 2.1% after the one-loop QCD correction is consistent with two-loop and running corrections not included here. The key result is that the $\overline{\text{MS}}$ prediction requires no QCD correction at all: it is 0.51% accurate at tree level.

5 Why the Top Does Not Hadronise

The top quark's confinement depth $\delta_t = 0.074$ is the smallest of all quarks [3]. The confinement time is:

$$\tau_{\text{conf}} \sim \frac{1}{\Lambda_{\text{QCD}}} \approx 3 \times 10^{-24}\,\text{s}. \tag{9}$$

The top quark lifetime is:

$$\tau_t \sim \frac{1}{\Gamma_t} \approx 5 \times 10^{-25}\,\text{s}. \tag{10}$$

Since $\tau_t \ll \tau_{\text{conf}}$, the top decays before hadronisation can occur.

In the bilateral picture: the top quark is so close to τ_0 — the junction of all bilateral ladders — that the crossing almost completes via the weak interaction before the colour structure has time to trap the shard in a prime gap. The top quark is the one quark whose shard nearly returns to zero before colour confinement claims it.

Remark 1. *The top quark Yukawa* $y_t \approx 1$ *is not a coincidence or a fine-tuning. It is the consequence of the top quark being the junction state at* τ_0*. The small departure from exactly 1 — the* 0.51% *displacement* $\delta_t = K_{\text{up}} - K_{\text{down}}$ *— is the bilateral asymmetry between the up-type and down-type colour sectors made physical. The top quark mass* is *the up/down asymmetry.*

6 The Complete Bilateral Ladder Picture

The full picture of the quark mass spectrum in the bilateral framework:

- **Top quark** ($\delta = 0.074$, junction state): sits at the base of all ladders, encodes the up/down asymmetry, decays before confinement.
- **Charm quark** ($\delta = 0.082$ from prime 5): the most free of the genuinely confined quarks. Nearly reached the prime at $p = 5$ (the tau prime). Heavy quark effective theory works well for charm.
- **Up quark** ($\delta = 0.297$ past prime 11): moderate confinement, just past the first generation prime.
- **Down quark** ($\delta = 0.474$ from prime 11): approaching prime 11 from below. The heaviest light quark.
- **Strange quark** ($\delta = 0.531$ past prime 7): straddles Λ_{QCD}, the most complicated light quark.
- **Bottom quark** ($\delta = 0.729$ past prime 3): the most confined heavy quark, deepest in the gap $[3, 5]$.

The top quark sits at the junction. Every other quark has committed to a position in a specific prime gap. The top has not. Its shard has not yet decided which ladder to climb.

7 Open Problems

1. Formal derivation of $\delta_t = K_{\text{up}} - K_{\text{down}}$. Theorem 1 states the result from the bilateral asymmetry argument. A formal derivation from the Yang–Mills action on $S^3 \times \mathbb{CP}^2$ — showing that the junction state displacement is precisely the Koide asymmetry — is not yet established.

2. The geometric origin of $(8\sqrt{5} - 17)/12$. The numerological reading ($8 = N_c \cdot \dim(S^3) - 1$, $17 = p_7$, $12 = N_c \cdot \dim(\mathbb{CP}^2)$) is suggestive but not yet a derivation.

3. Two-loop correction. The one-loop $\overline{\text{MS}}$–pole conversion leaves a 2.1% residual. Whether the two-loop correction accounts for this precisely has not been computed.

8 Conclusion

The top quark mass in the $\overline{\text{MS}}$ scheme is:

$$m_t(\overline{\text{MS}}) = \frac{v}{\sqrt{2}} \exp\left(-\frac{8\sqrt{5} - 17}{12}\right) = 161.68\,\text{GeV}, \tag{11}$$

within 0.51% of the observed value, with no free parameters.

The formula follows from the top quark being the junction state at τ_0 — the bilateral crossing point where the up-type and down-type colour ladders meet. Its displacement from τ_0 is the asymmetry between the two quark sector Koide values: $\delta_t = K_{\text{up}} - K_{\text{down}}$.

The top quark mass is not set by a prime exponential. It is set by the golden ratio — through the asymmetry $K_{\text{up}} - K_{\text{down}} = 4/(3\varphi) - 3/4$. The golden ratio enters the top quark mass because the bilateral self-similarity constant φ governs the asymmetry between the two quark Koide sectors [2].

References

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[3] D. Low, *Confinement as Incomplete Bilateral Crossing*, preprint (2025), `ontologia.co.uk/`.

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[6] Particle Data Group (S. Navas et al.), *Review of Particle Physics*, Phys. Rev. D **110** (2024) 030001.